Diffraction-grating VCN interferometry and Its Applications

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Workshop on
"UCN/VCN sources at the Institute of Nuclear Physics (Kazakhstan) and their applications"

MLZ is a cooperation between:

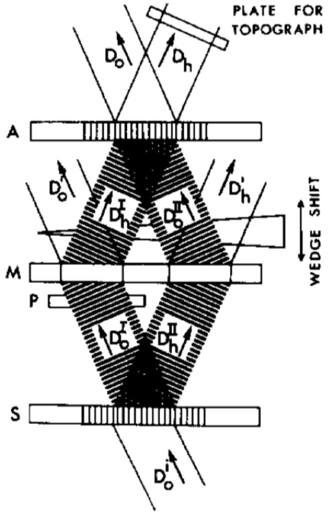
Almaty, April 8 – 11, 2024

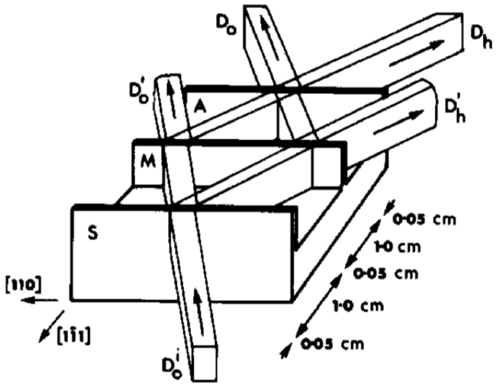
Contents:

1. VCN diffraction grating interferometer.
2. Applications to neutron charge quest – lowering the experimental limit by orders of magnitude.
3. High-resolution Neutron Speed-Echo spectroscopy of VCN.

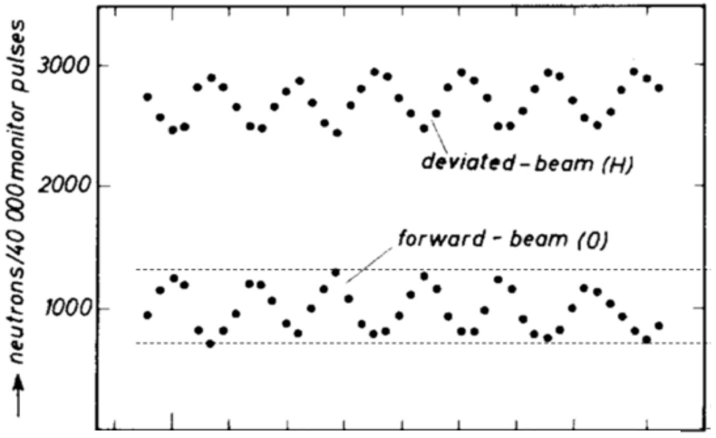
Perfect crystal neutron interferometer

Perfect crystal neutron LLL interferometer





Introduced phase shift $(\Delta\varphi)$ between beams in the NI



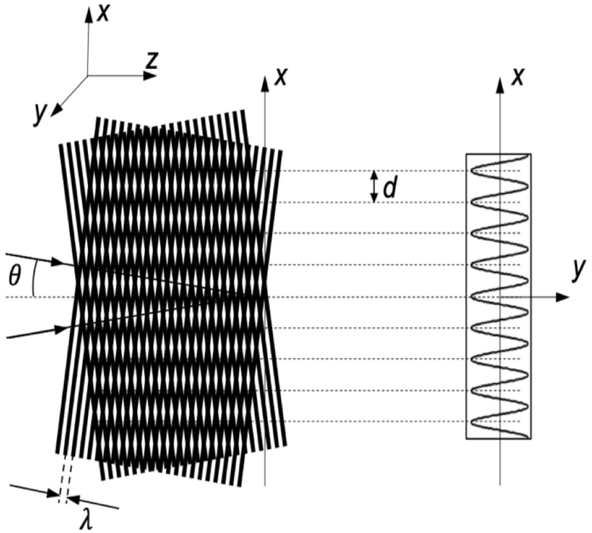
Phase shift $(\Delta\varphi)$ between beams in NI

$$I_H(\Delta\varphi) = B - A\cos(\Delta\varphi)$$

Swapping intensity between O- and H-beams

$$I_O(\Delta\varphi) = V(1 + \cos(\Delta\varphi))$$

H. Rauch et al., 1974



Alternatively:

- crossing coherent waves produce interference pattern of period d
- these interference fringes are superimposed with crystal lattice (d) => Moiré effect (fringes)
- $\Delta\varphi$ results in the lateral shift of interference fringes w.r.t. crystal lattice
- this leads to oscillations of Moiré fringes, i.e. oscillations of recorded intensity

(Very) cold neutrons: coherent beam splitting

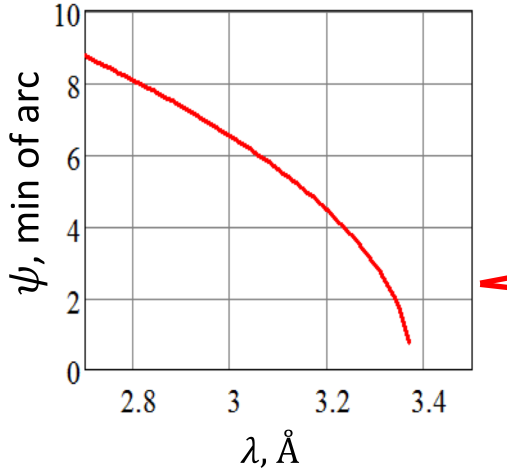
For cold neutrons one should employ other than Laue diffraction coherent splitting of neutron waves: diffraction on periodical structures (gratings) or reflection from semi-transparent coatings.

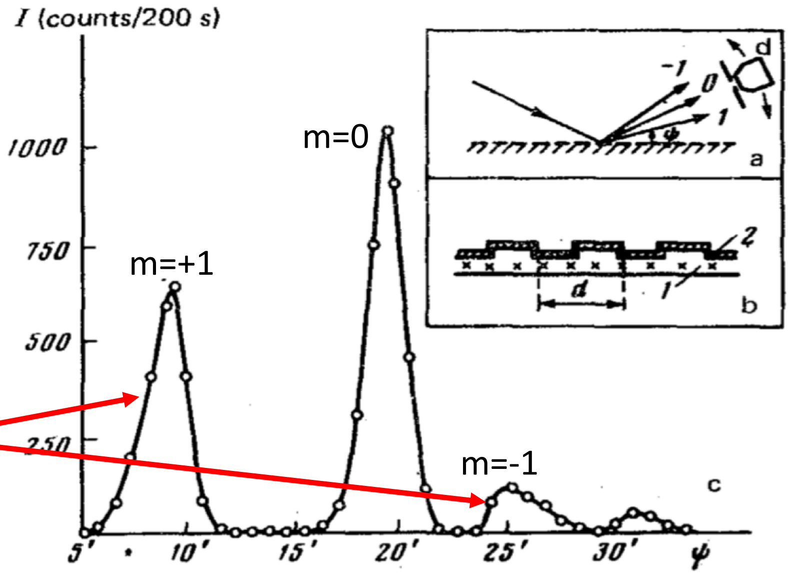
Effective neutron diffraction gratings: modulated surface relief

A.Ioffe et al, JETP letters 33, 374 (1981)

$d = 21\ \mu\text{m} \quad \lambda = 2.7\ \text{Å}, \quad \Delta\lambda/\lambda = 32\%$ (Ni mirror)

Diffraction efficiency:
m=+1 ≈ 5%
m=0 ≈ 7.5%



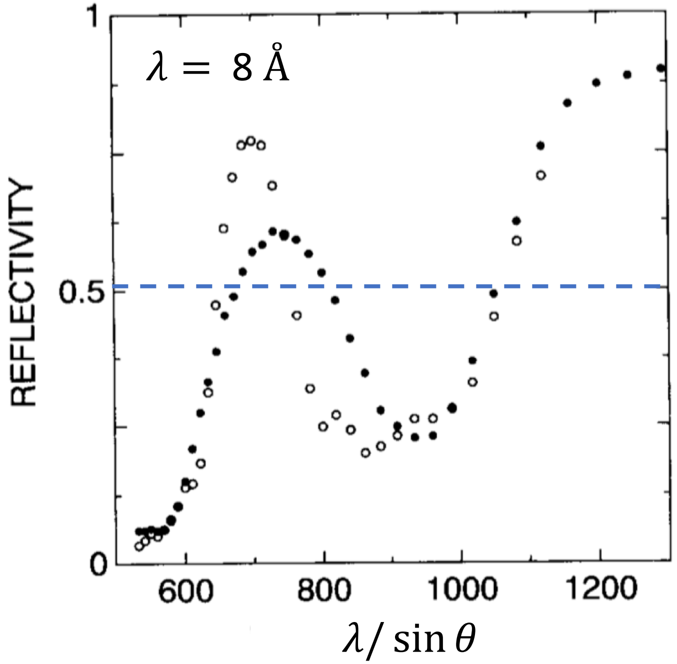


Note asymmetry: spectrum of incident beam => spectroscopy

Coherent beam splitter

*T. Ebisawa et al, NIM A **344**, 597 (1994)*

V-Ti multilayer mirror with spacing d=360 Å



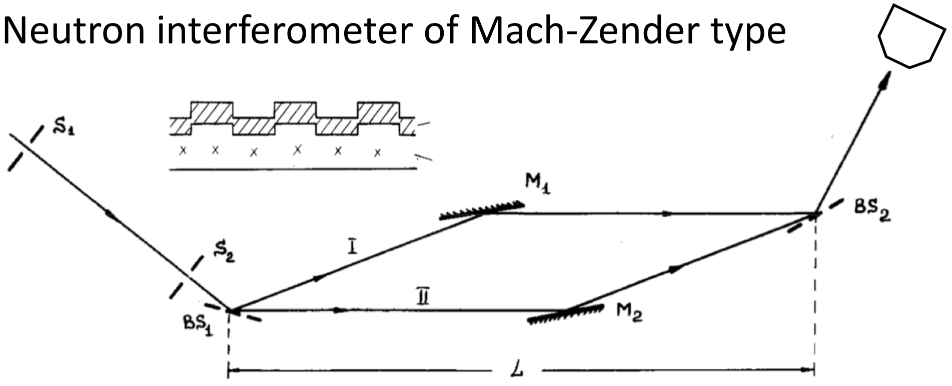
Cold neutron interferometers

Volume 111, number 7 PHYSICS LETTERS 30 September 1985

TEST OF A DIFFRACTION GRATING NEUTRON INTERFEROMETER

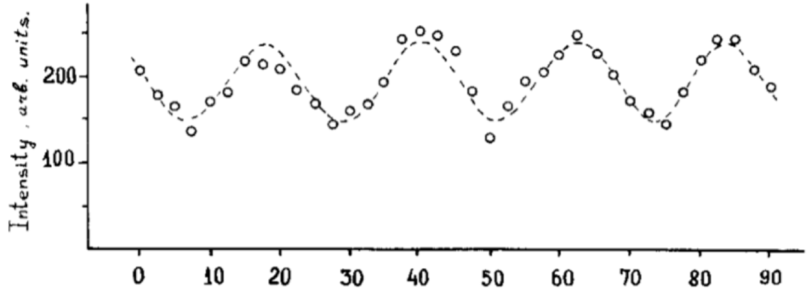
A.I. IOFFE, V.S. ZABIYAKIN and G.M. DRABKIN

Neutron interferometer of Mach-Zender type



Moire pattern with a period d :

$\lambda = 3.15$ Å $d = 21\ \mu$m



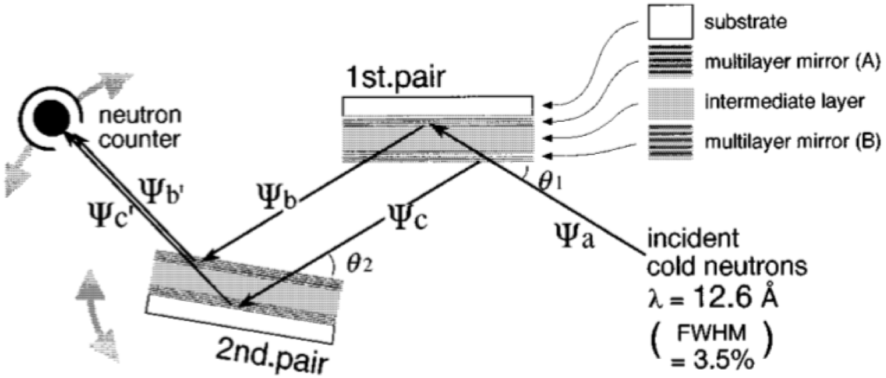
Lateral shift of BS_2, μm

PHYSICAL REVIEW A VOLUME 54, NUMBER 1 JULY 1996

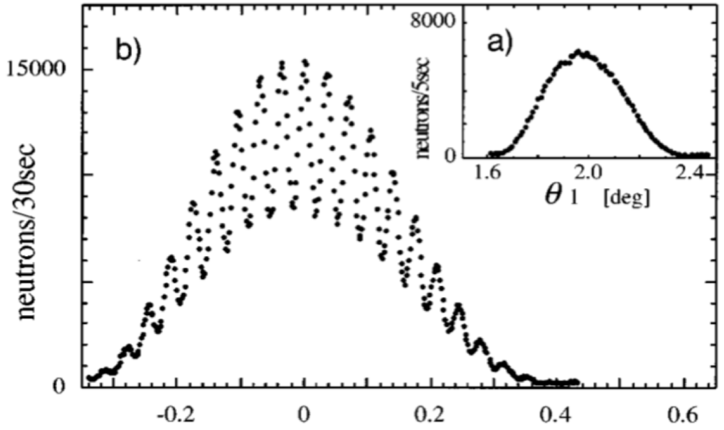
Interferometer for cold neutrons using multilayer mirrors

Haruhiko Funahashi,[1,*] Toru Ebisawa,[1] Tomohito Haseyama,[2] Masahiro Hino,[3] Akira Masaike,[2] Yoshié Otake,[4]

Idea: T. Ebisawa et al, NIM A ***344****, 597 (1994)*

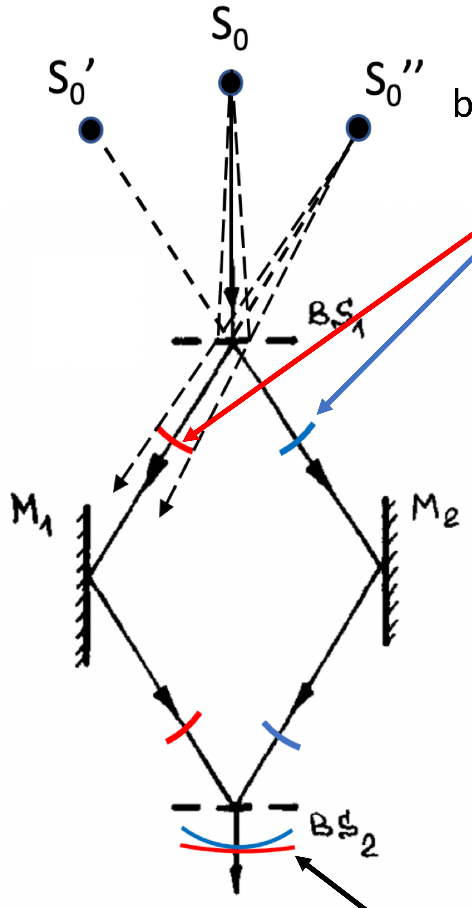


See also presentation of H. Shimizu at 1st UCN/VCN workshop

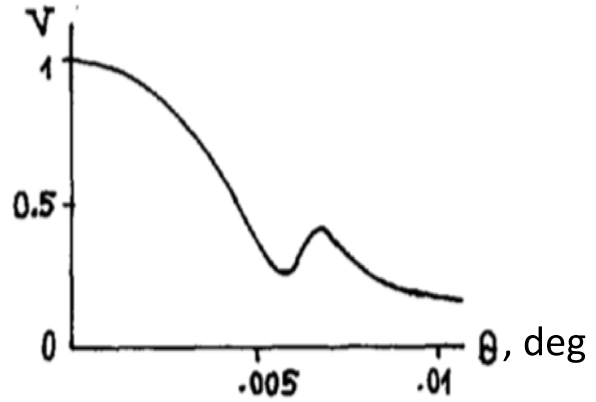


Angle between first and second pairs

3- grating interferometers



Spherical incident wavefronts diffracted by periodic structures are principally aberrated, and non-identical for m=1 and m=-1.



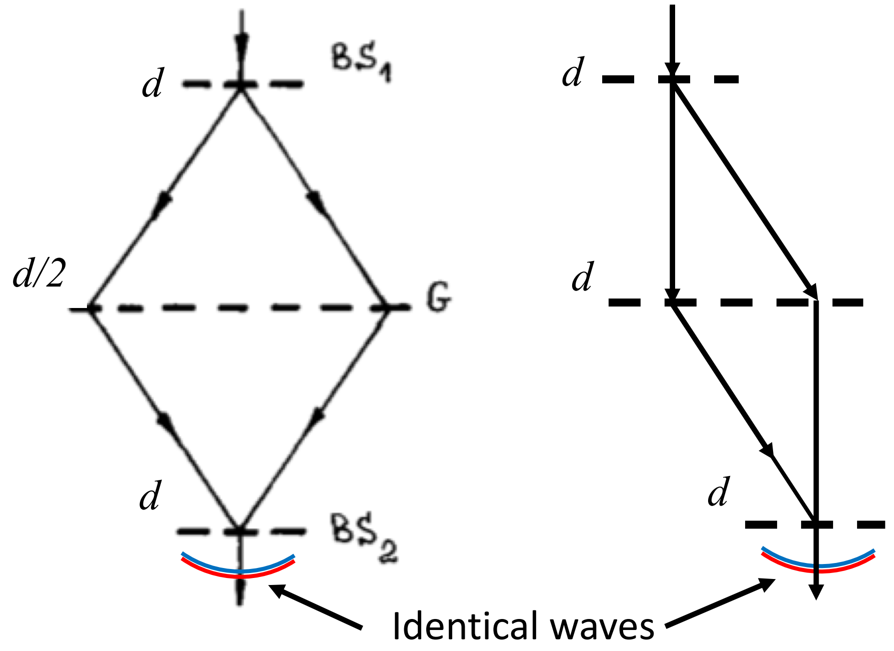
=> Strong requirements to incident beam divergence
⇒ Bad for neutrons in general; unfeasible for VCN

Interference of two non-indentical waves:
=> non-constant period of the interference pattern
=> amplitude modulation over the beam cross-section
=> low visibility V

Unavoidable different aberrations in interfering beams:
=> add complimenting aberrations for equalization.

Deflection --> Diffraction: gratings instead of mirrors

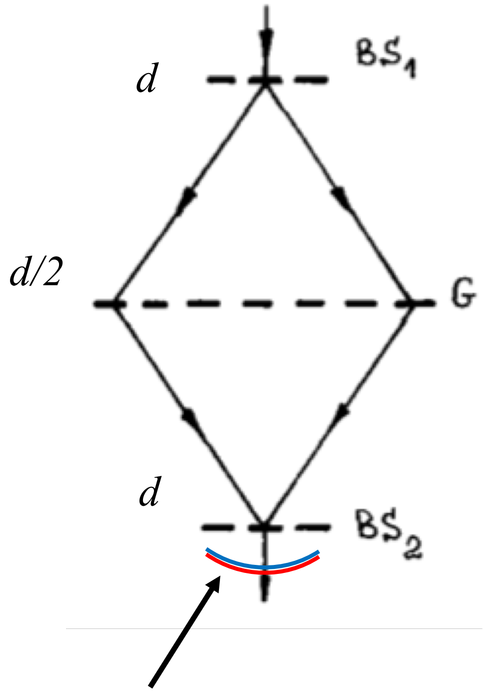
3-grating interferometers



Aberration analysis shows that now interfering wavefronts are distorted identically and V=1:
=> no requirements to incident beam divergence

A.Ioffe, Physica B 174 (1991) 385.

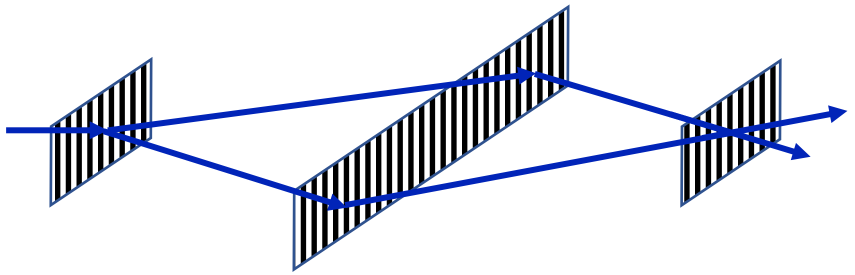
Diffraction grating interferometers



This is not the Talbot interferometer: Talbot effect is a near-field diffraction effect, where the self-imaging of periodic objects (gratings) **requires spatially coherent illumination.**

Here: the imaging of a grating by a second grating **regardless of the coherence of the source.**

- First shown by first-order diffraction theory (i.e. without accounting for aberrations): (*B.Chang, R.Alferness, E.Leith (Apll. Opt. 14 (1975) 1569*) .
- Aberration analysis (higher-orders diffraction theory): full compensation of aberrations => interfering waves are identical (*A.Ioffe, NIM A268 (1988) 169*).



Such interferometer works regardless of the source coherence, i.e. for non-monochromatic and non-collimated neutron beam!

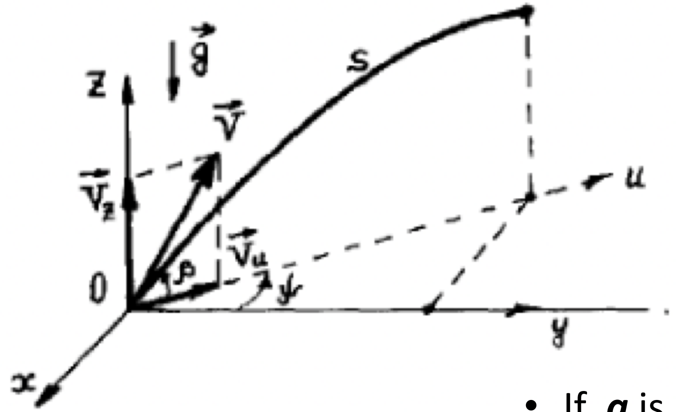
Transition to neutrons:
refraction index of vacuum in gravitational field ≠ 1.
(*I.M Frank, A.I Frank, JETP Lett.* ***28*** *(1978) 515*)

$$n = \sqrt{1 - 2gz\left(\frac{m_n}{h}\right)^2 \lambda^2}$$

=>

As neutrons propagate on parabolic trajectories, vacuum has non-linear refraction index. This is not trivial, will be discussed later.

VCN grating interferometers in gravitational field

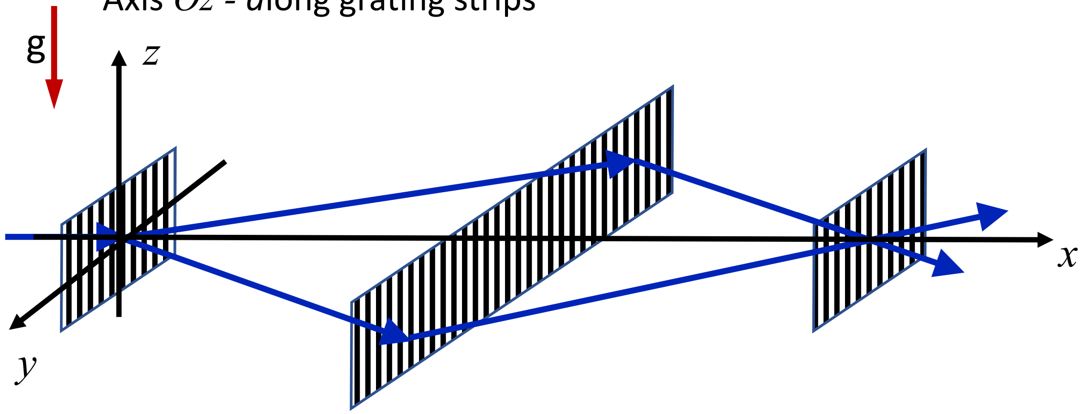


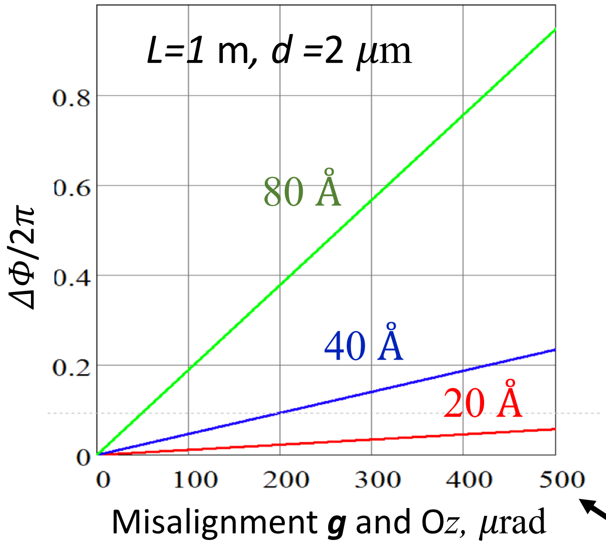
$$\Phi(u)=\frac{m_n}{\hbar}\left[uV_u+\frac{V_z^3}{3g}-\frac{V_u^3}{3g}\left(\tan\beta-\frac{gu}{V_u^2}\right)^3\right],$$

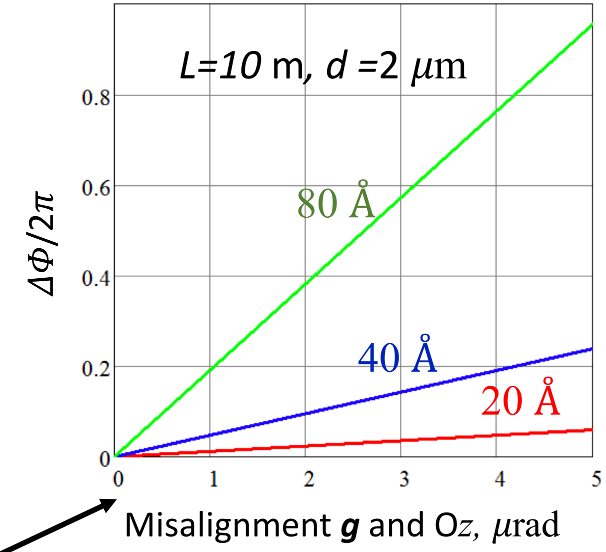
Calculating the velocity components immediately after diffraction, it is possible to calculate the phase shift of neutron wave during its following propagation.

- If ***g*** is strictly parallel to O*z* (grating strips), gravitational potentials for both sub-beams are equal (symmetry).
- Violation of this symmetry leads to neutron trajectories rising to different heights => phase difference.

Axis *Oz* - *a*long grating strips







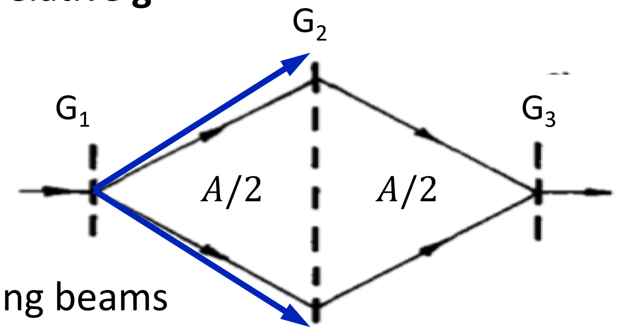
Note different scales!

A challenge for the use of VCN !

3-grating interferometer

(+) for VCN: aberration-free, V=100% for full incoherent illumination
(-) for VCN: requires $\mu\mathrm{rad}$ alignment relative **g**
(-) parasitic Sagnac effect

$$\varphi_S = \frac{2m_n}{\hbar}(\boldsymbol{\omega}\cdot\boldsymbol{A}) = \frac{2m_n}{\hbar}\omega_0 A\sin\theta_1,$$



$A = \frac{\lambda}{d}\frac{L}{2}$ - area enclosed by interfering beams

Problem is not φ_S, rather $\Delta\varphi_S$ because of different A

Scatter in λ: $\Delta\lambda \rightarrow \Delta A \rightarrow \Delta\varphi_S$ $\qquad \Delta\varphi_S = \varphi_S\frac{\Delta\lambda}{\lambda} = 0.1A$

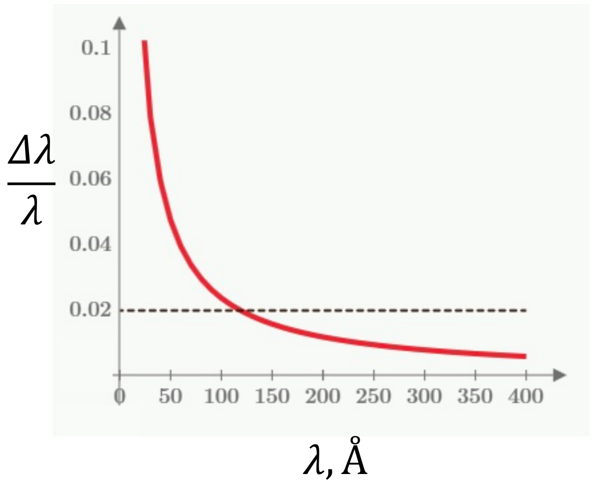
ω_0 = 7.29 $\cdot 10^{-5}$ s^{-1} angular velocity (Earth's rotation)

$\theta_1 \approx 43°$ - latitude angle (Almaty)

Interference fringes are washed out for $\Delta\varphi_S > \pi/2$:

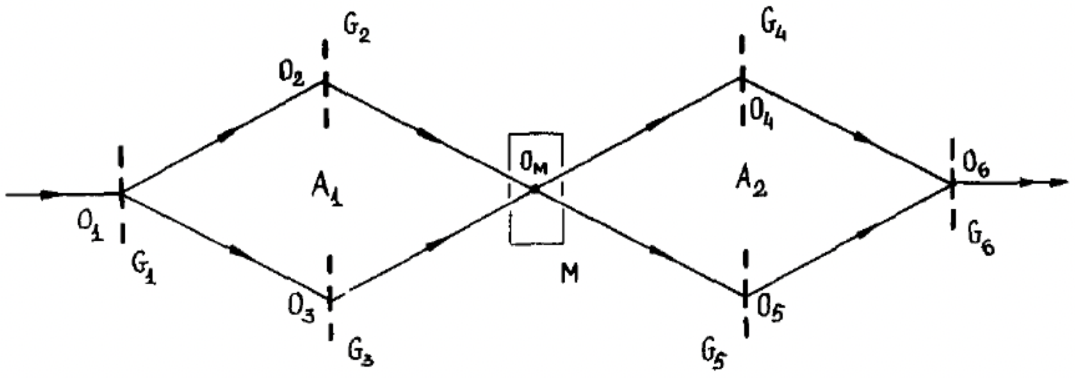
=> limitation on $\frac{\Delta\lambda}{\lambda}$

Not feasible with VCNs.



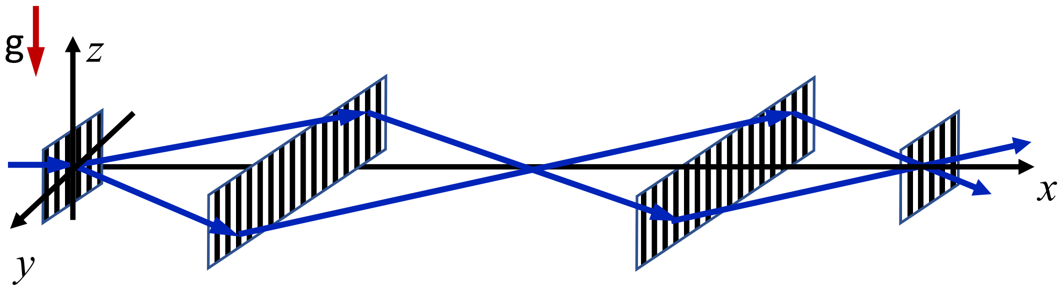
4-grating interferometer

A.Ioffe, NIM A268 (1988) 169.



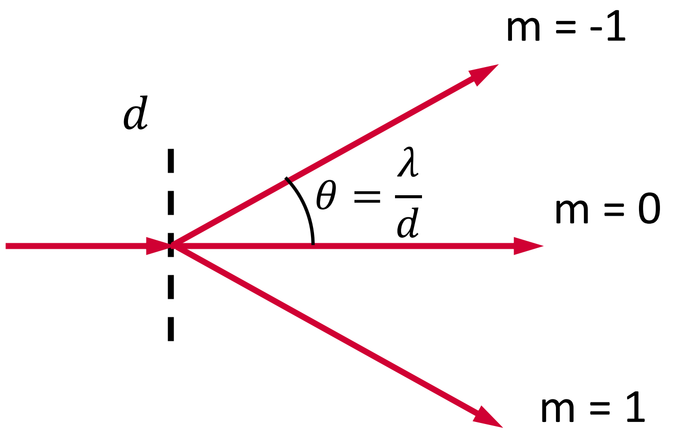
Symmetric scheme:

- A$_1$= - A$_2$ - complete compensation of Sagnac effect
- complete compensation of gravitational phase difference



Not for free: one more grating - additional intensity losses

Diffraction gratings for VCNs



Requirements: small period and high diffraction efficiency

- Photolithographic gratings (stamping in photoresist)
- Holographic photolithographic gratings (interference lithography)
- Holographic nanodiamond-polymer composite gratings (E.Hadden et al, Appl. Phys. Lett. 124, 071901 (2024))

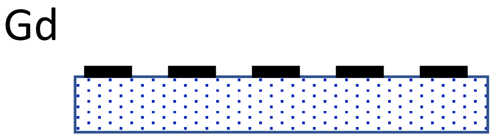
Phase gratings

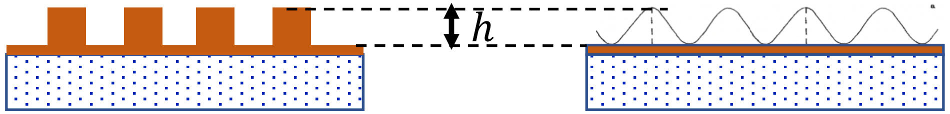
Phase shift: $\varphi = \frac{2\pi\lambda}{\rho} h$

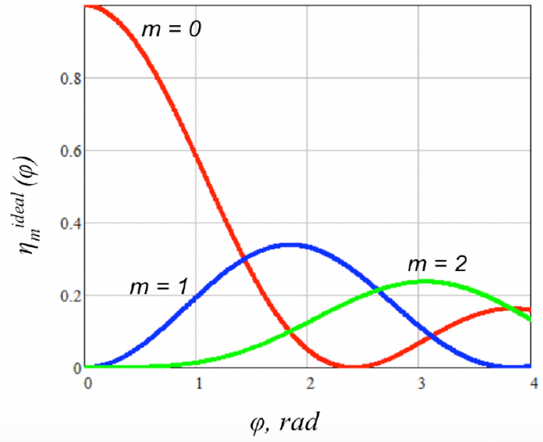
ρ - scattering length density

h = 1.74 µm for λ =20 Å.

Amplitude gratings







Diffraction efficiency:

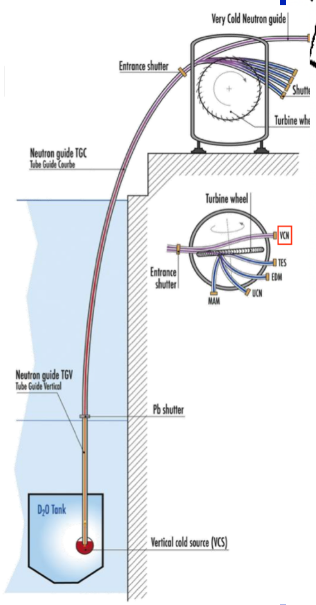
Amplitude gratings	Phase gratings	
$\eta_m = \frac{1}{\pi^2 m^2}$	$\eta_m = \frac{4}{\pi^2 m^2} \sin^2(\varphi)$	$\eta_m = [J_m(\varphi)]^2$
η_1^{max} = 10.1%	η_1^{max} = 40.4%	η_1^{max} = 33.8%

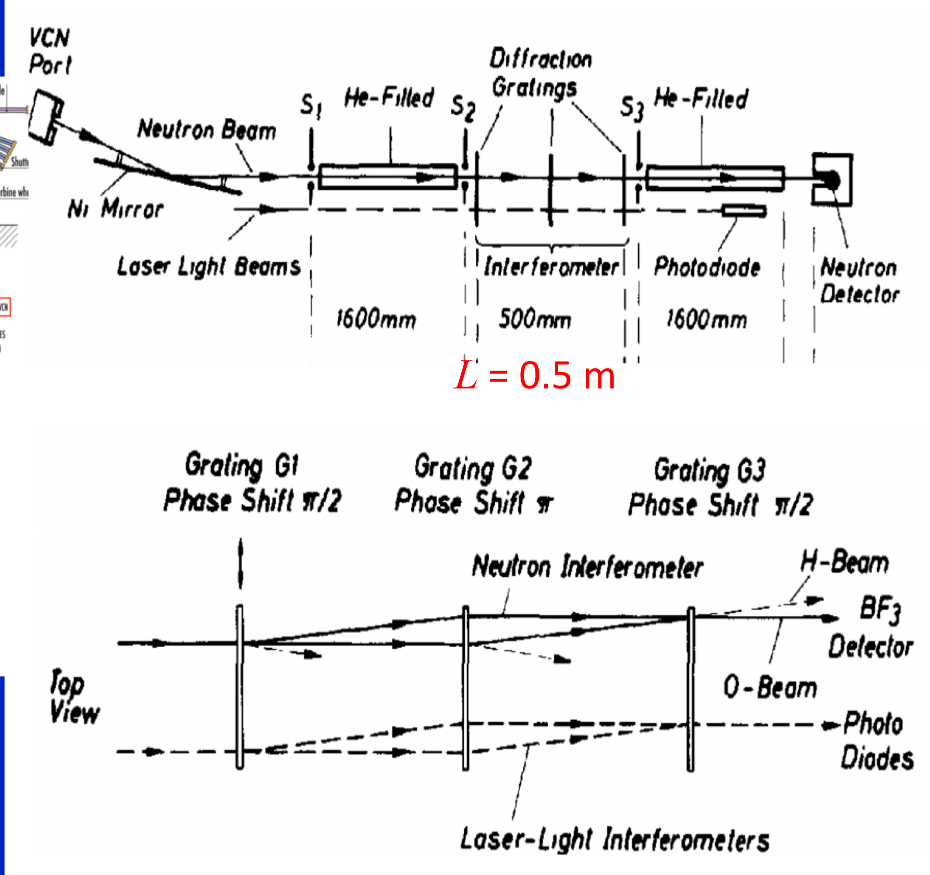
First realization of VCN diffraction grating interferometer

Volume 140, number 7,8 PHYSICS LETTERS A 9 October 1989

A PHASE-GRATING INTERFEROMETER FOR VERY COLD NEUTRONS

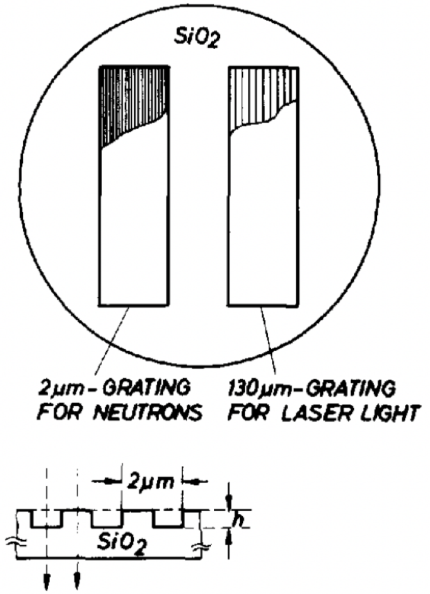
M.Gruber, K.Eder, A.Zeilinger, R.Gähler, W.Mampe





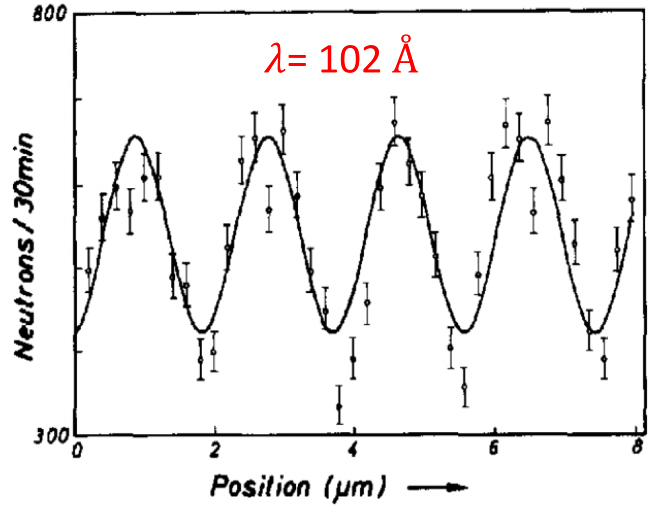
L = 0.5 m

Phase diffraction gratings:



$d = 2\ \mu$m

Interference pattern: obtained by translation of G_1 along its surface.



JÜLICH Forschungszentrum

Potential application of VCN interferometry: searching for the net electric charge of neutrons

- I will not discuss “why” to measure q_n, just “how” to measure it.
- Earlier attempts and current experimental limit on q_n .
- VCN interferometry
- Utilizing VCN to improve the experimental limit on q_n .

Previous experiments and limits on neutron charge q_n

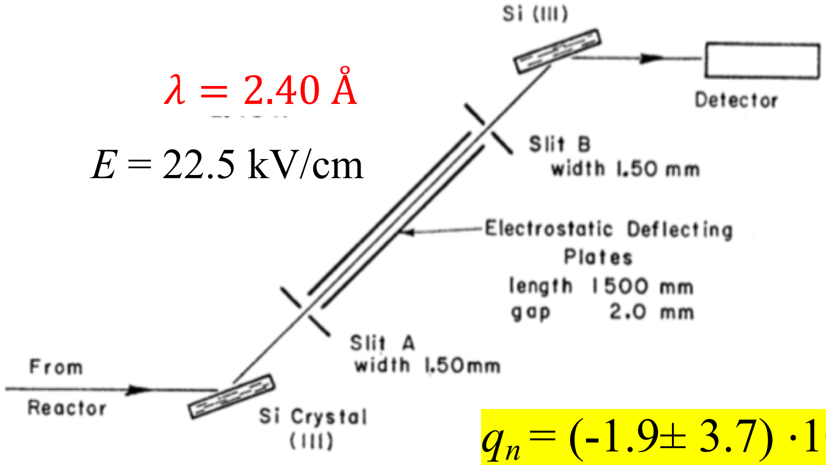
PHYSICAL REVIEW VOLUME 153, NUMBER 5 25 JANUARY 1967

Experimental Limit for the Neutron Charge*

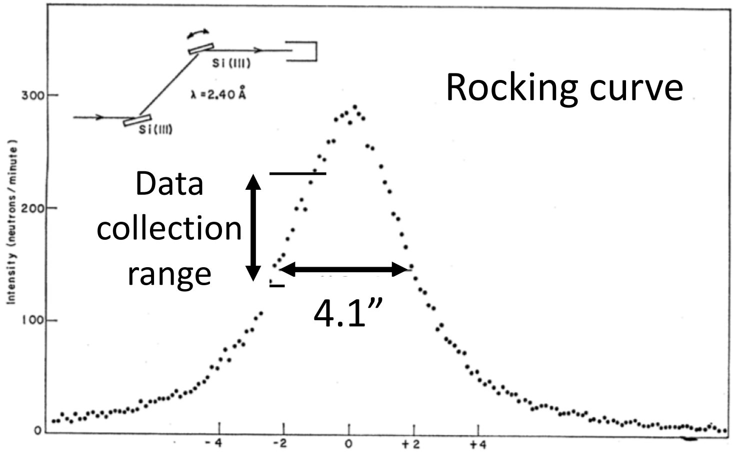
C. G. Shull, K. W. Billman, and F. A. Wedgwood†

$\lambda = 2.40$ Å

$E = 22.5$ kV/cm



$q_n = (-1.9 \pm 3.7) \cdot 10^{-18}\, e$ (CI 68%)

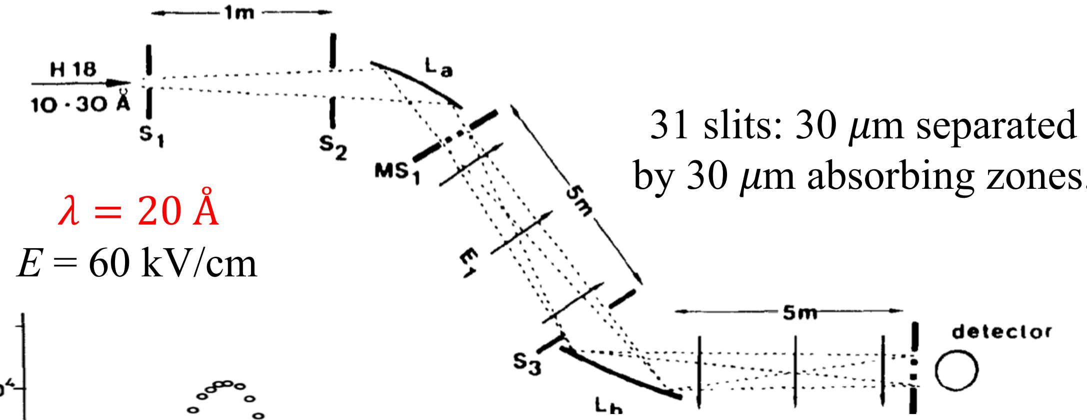


Rocking angle (sec of arc)

PHYSICAL REVIEW D VOLUME 25, NUMBER 11 1 JUNE 1982

Experimental limit for the charge of the free neutron

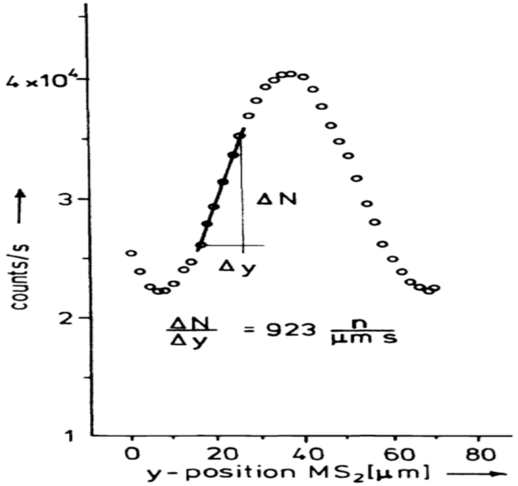
R. Gähler, J. Kalus, W. Mampe

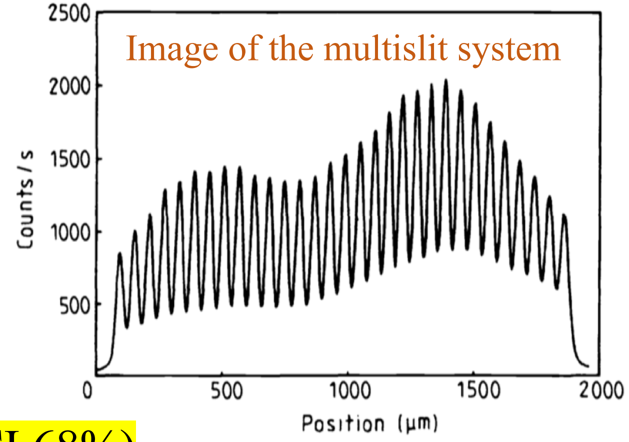


31 slits: 30 μm separated by 30 μm absorbing zones.

$\lambda = 20$ Å

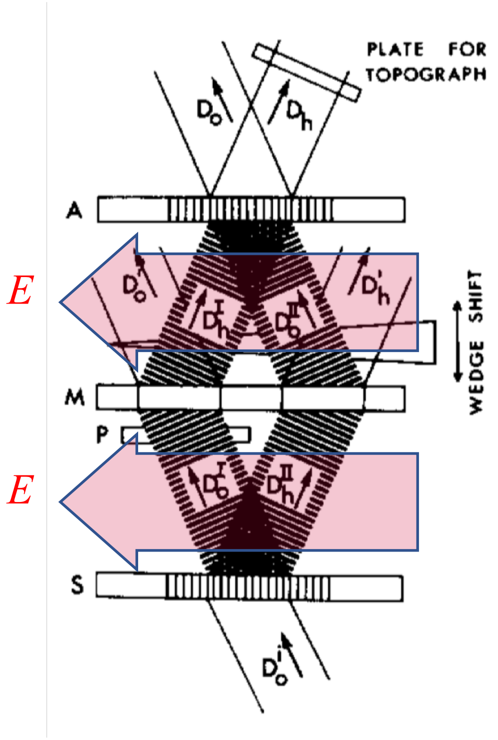
$E = 60$ kV/cm





$q_n = (-0.4 \pm 1.1) \cdot 10^{-21}\, e$ (CI 68%)

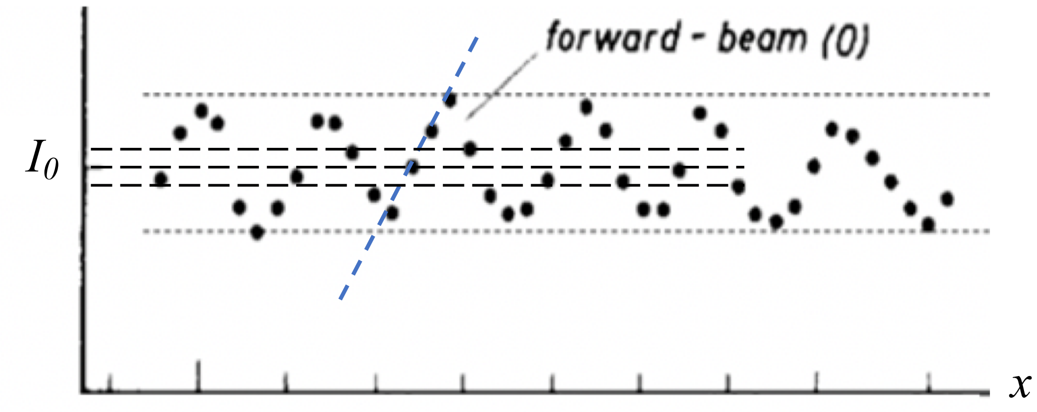
Gedanken experiment with crystal interferometer: neutron charge q_n



Electric field across neutron beams => shift of interference pattern:

$$\Delta x = \frac{1}{2} qE \left(\frac{L}{v}\right)^2 = \frac{1}{2} qE \left(\frac{L}{h} m\lambda\right)^2 \sim qEL^2\lambda^2$$

$$I(\Delta x) = I_0 V \left(1 + \cos\frac{2\pi}{d}\Delta x\right)$$ --> Shift by d (lattice spacing) corresponds to full intensity oscillation

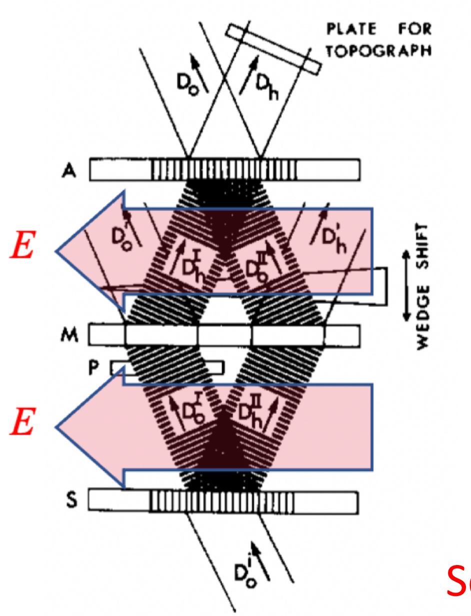


$$q = \frac{\sqrt{2}\, d}{\pi \sqrt{I_0}} \frac{1.5}{V\ E\ L^2\ \lambda^2} \left(\frac{h}{m}\right)^2$$

For E = 60 kV/cm, L = 5cm, λ=2 Å, d=1.92 Å $q_n \geq 3 \cdot 10^{-20}\ e$ (CI 90%) in 100 days

Important: $q \sim \frac{d}{\sqrt{I_0} E\ L^2\ \lambda^2}$ - this is a kind of FOM

Neutron interferometer with larger length and wavelength: VCN



$$q_n \sim \frac{d}{\sqrt{I_0}\, E\, L^2\, \lambda^2}$$

Now imagine we can modify our interferometer towards larger length and wavelength, however with corresponding increase of d.

Scaling to VCN:

λ: 2 Å -> 20 Å => x10^2 L: 5 cm -> 5 m => x10^4	x	d: 2 Å -> 1 μm => x $(2 \cdot 10^{-4})$ I_0 : $\sqrt{\lambda^{-5}}$ => x $(3 \cdot 10^{-3})$	=>	Thermal to cold neutron source: x15

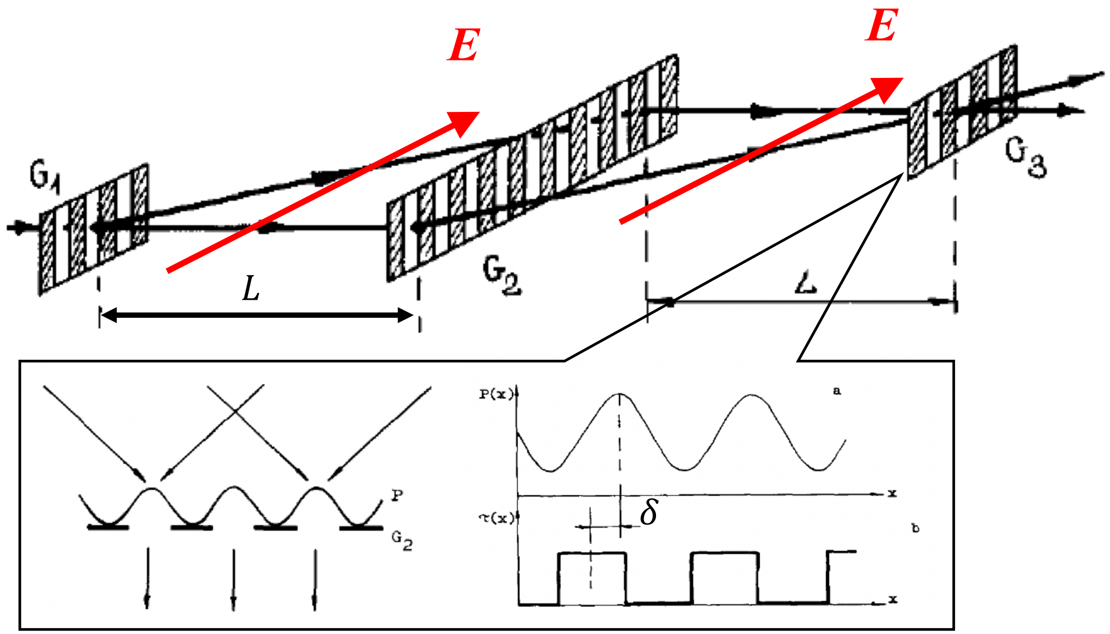
Total gain about 10 => one can put a harder limit on q_n

However, for cold neutrons one should use other than Laue diffraction coherent splitting of neutron waves.

VCN diffraction grating interferometer for search of q_n

Electric field applied across interferometer beams

A. Ioffe, NIM A228 (1984) 141; NIM A268 (1988) 169.



$$\delta = \frac{1}{2} qE \left(\frac{L}{h} m\lambda\right)^2 \qquad q = \frac{\sqrt{2}\, d}{\pi \sqrt{I_0}} \frac{1.5}{E\, L^2 \lambda^2} \left(\frac{h}{m}\right)^2$$

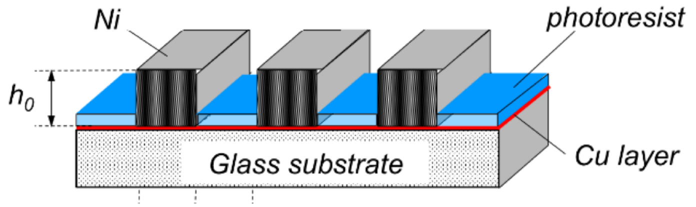
$$I(\Delta x) = I_0 V \left(1 + \cos\frac{2\pi}{d}\delta\right)$$

1987- proposal to ILL (accepted, but was not materialized): to use the same setup at H18 as for previous q_n experiment:

I_0 = 200 n/s , λ= (20 ±0.15) Å, E =60 kV/cm, L =5 m

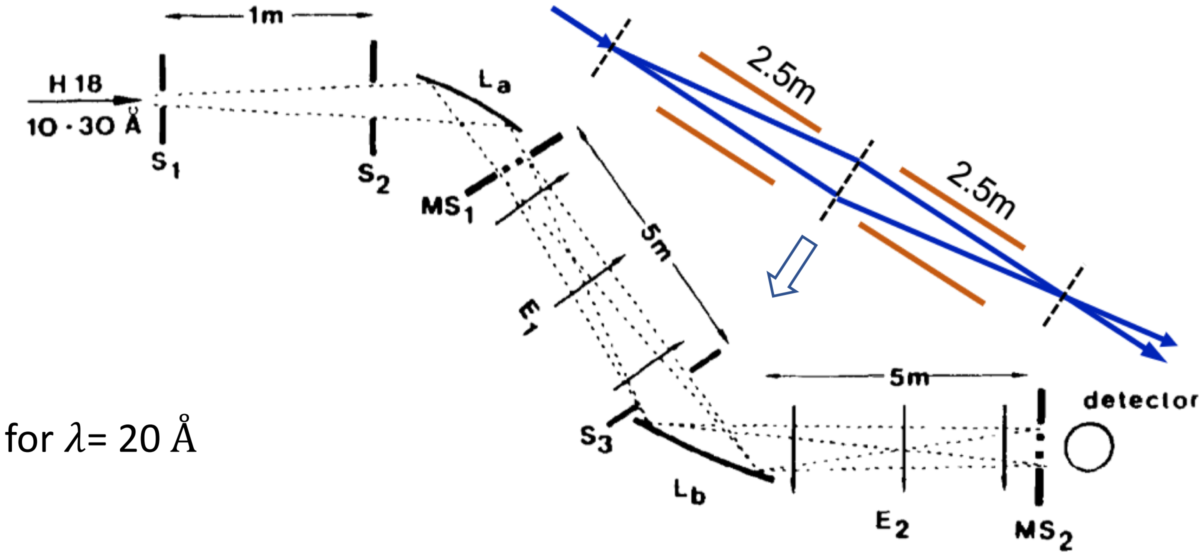
$q_n \geq 2 \cdot 10^{-22}\, e$ in 60 days - order of magnitude improvement

Phase diffraction gratings: surface relief



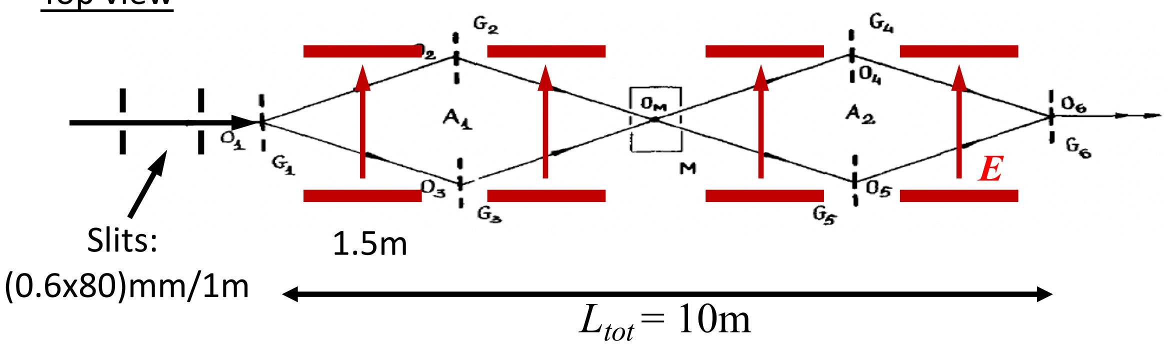
d = 3.3 μm

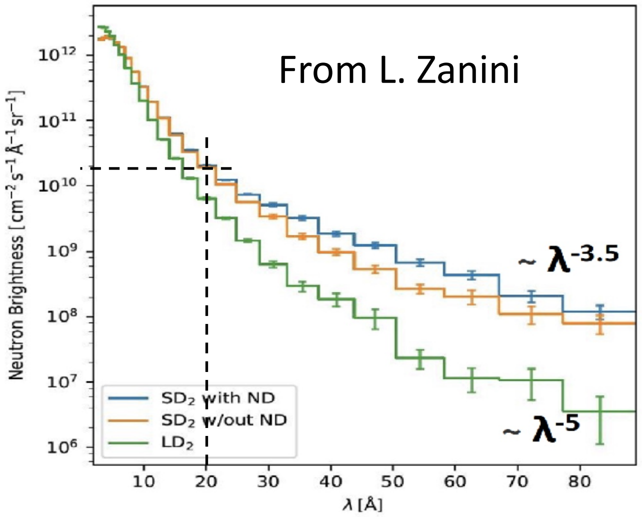
h_0 =1.7 μm: phase shift π for λ= 20 Å



VCN diffraction grating interferometer at ESS: search for q_n

Top view





B (20 Å)= 2 x10^{10}

Expected counting rate:

$I_{rec}(20\ \text{Å}) \approx 4.9 \cdot 10^3$ n/s

$d = 2\ \mu m$

V=50%

E =60 kV/cm

L_E =6m

Beam parameters: the same as at the ILL setup

Beam cross-section: S_{beam} = 0.48 cm^2

Solid angles: ω_x = 0.0006

ω_y = 0.08 (L=10m, last slit is G_4)

$\Delta\lambda = 3956 \cdot T_{rep}/L_{source-det}$ =14 Å

$$q(\lambda) = \frac{\sqrt{2}\, d}{\pi \sqrt{I_{rec}(\lambda)\ 8.64\ 10^4\ N_{days}}} \frac{1.65}{V\ E\ L^2\ \lambda^2} \left(\frac{h}{m}\right)^2$$

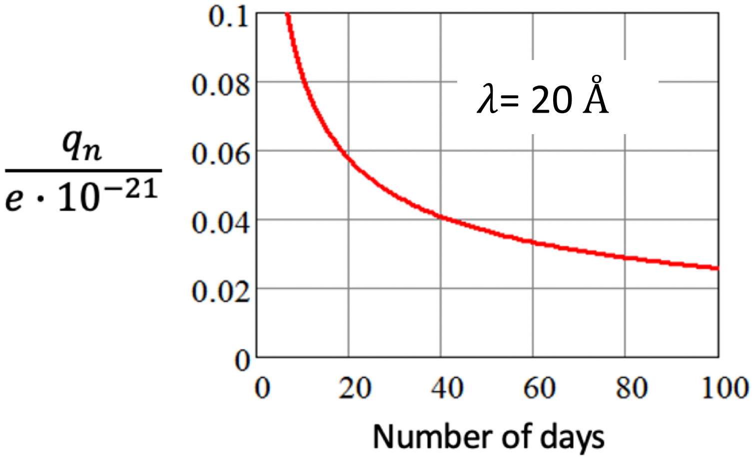
Diffraction efficiency: η^4 =0.008 (η = 30%)

Transmission of substrates (λ= 20 Å) :

Si 4 x 0.07 cm: T_{Si}= 0.93

SiO_2 4 x 0.3 cm: T_{SiO2} = 0.63

$$I_{rec}(\lambda) = B(\lambda)\ S_{beam}\ \omega_x\ \omega_y\ \eta^4\ \Delta\lambda\ T_{SiO2}$$



$q_n \geq 3 \cdot 10^{-23}\ e$ in 80 days (CI 90%)

2 orders of magnitude better, than the present day limit

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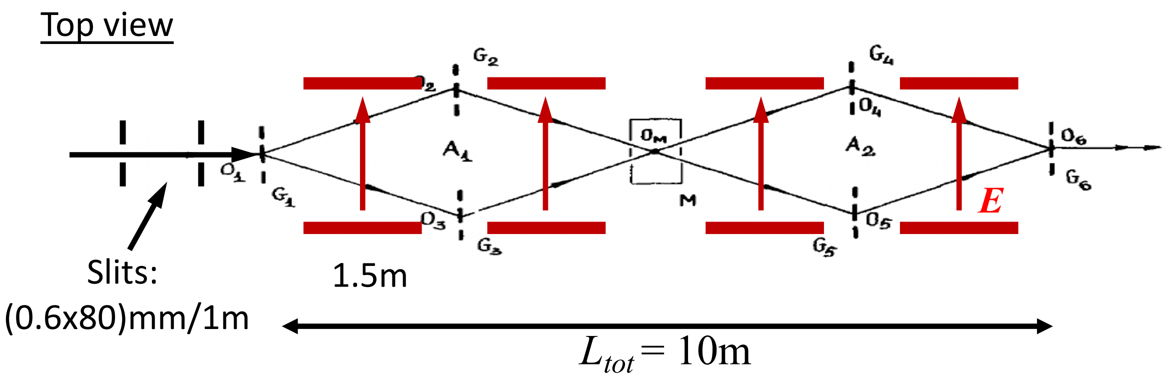
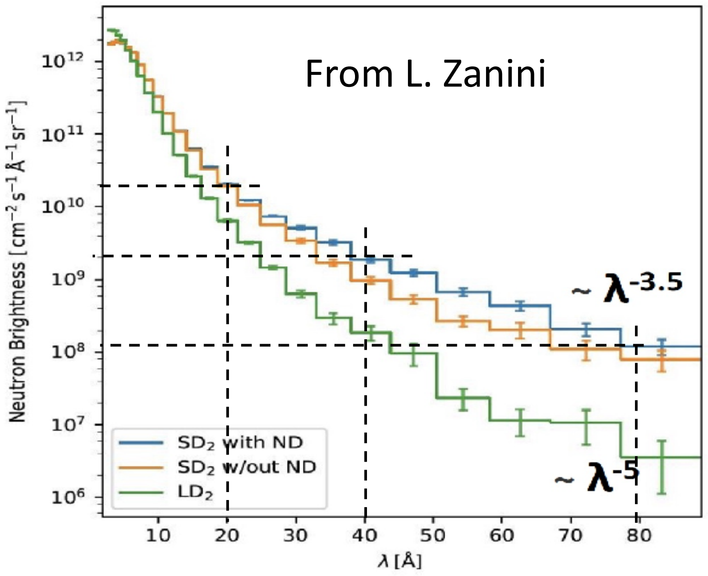
VCN diffraction grating interferometer at ESS: search for q_n





Expected counting rate:

$I_{rec}(20\ \text{Å}) \approx 4.9\cdot10^3$ n/s

$I_{rec}(40\ \text{Å}) \approx 4.9\cdot10^2$ n/s

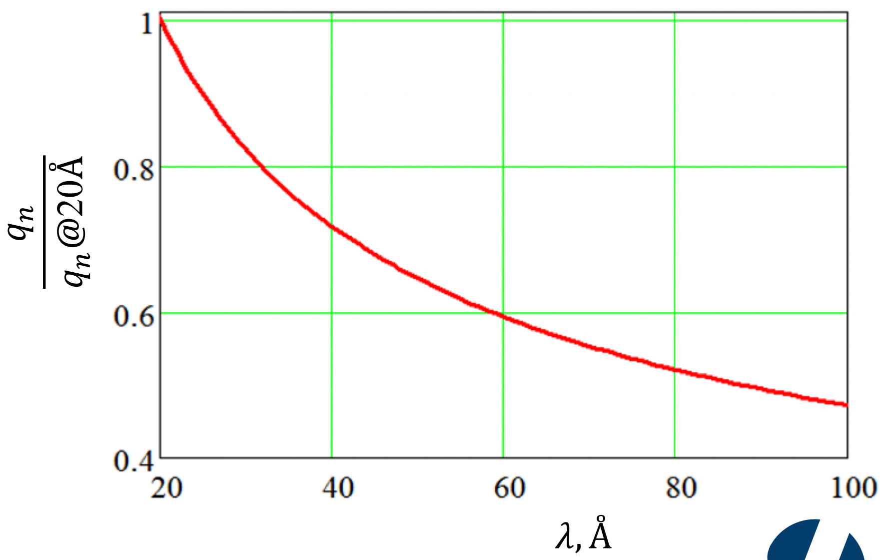
$I_{rec}(80\ \text{Å}) \approx 4\cdot10^1$ n/s

Transition to higher λ (> 20Å): does it make sense?

$$q_n \sim \frac{d}{\sqrt{I_0}E\ L^2\ \lambda^2}$$

LD$_2$: $q_n \sim \dfrac{1}{\lambda^{-2.5}\ \lambda^2} \sim \lambda^{0.5}$ Getting worse

SD$_2$: $q_n \sim \dfrac{1}{\lambda^{-1.75}\ \lambda^2} \sim \lambda^{-0.25}$ Getting better



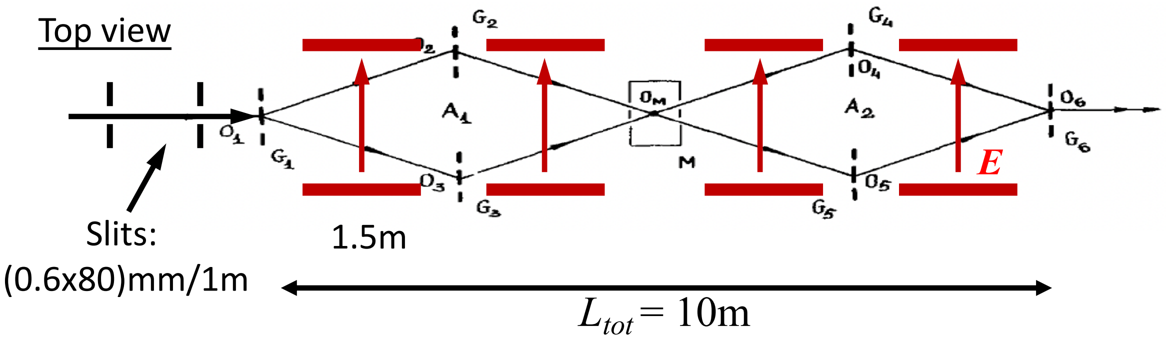
SD_2 with ND is a game changer:

Transition from 20 Å to 80 Å gives factor of 2 improvement:

$q_n \geq 1.5 \cdot 10^{-23}\ e$ in 80 days (CI 90%)

Cold source at WWR-K?

Potential for further improvements in search for q_n



$$q_n \sim \frac{d}{\sqrt{I_0}\, E\, L^2\, \lambda^2}$$

Practically, only “free” parameter is grating period d .

Reducing the period d of diffraction gratings to sub-μm:

$\Rightarrow$ direct gain as $q_n \sim d$

$\Rightarrow$ increase of diffraction angle $\theta = \frac{\lambda}{d}$,

therefore gain in incident beam intensity $\sim d^2$:

gain in solid angle $\omega_x \sim d$ (still $\ll \lambda/d$)

gain in beam cross-section $\sim d$

Overall gain in $q_n \sim d^{-2}$

V=50%, E =60 kV/cm, L_E =6m

Beam parameters:

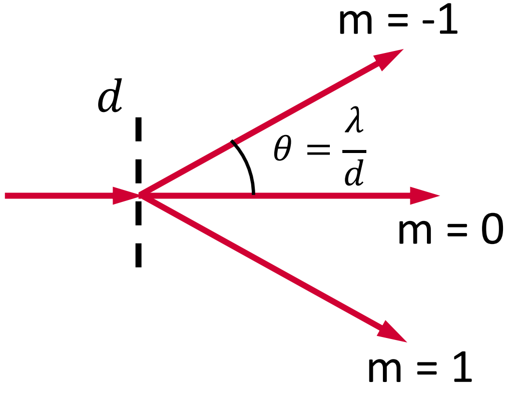
Beam cross-section: S_{beam} = 0.48 cm^2

Solid angles:

ω_x = 0.0006 << λ/d = 0.005

ω_y = 0.08 (L=10m, last slit is G_4)

Band: $\Delta\lambda$=14 Å

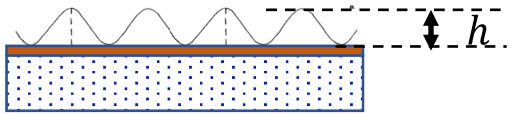


d: 2 μm --> 0.5 μm results in additional improvement by an order of magnitude: $q_n \geq 10^{-24}\, e$

$\Rightarrow$ Holographic (interference) gratings

$\Rightarrow$ Holographic ND-composite gratings

(*E.Hadden et al, Appl.Phys.Lett., 2024*))



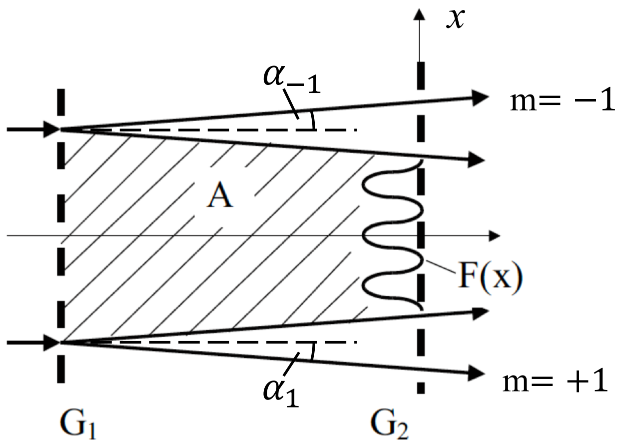
h = 0.44 µm for λ =80 Å

Another potential application for VCN:
very high resolution Neutron Speed Echo spectroscopy (NSPE)

Neutron Speed Echo (NSPE): principle

Two stationary diffraction gratings

A. Ioffe, in Neutron Spin Echo Spectroscopy, Lecture Notes in Physics (2002) p.142.



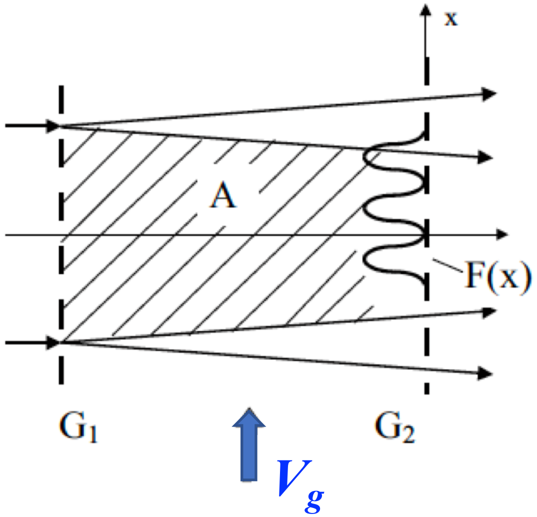
Diffraction on $G1$:

- amplitude division of wavefronts
- diffracted waves are coherent
- in superposition region: sinusoidal amplitude distribution $F(x)$ with period d_F

$$d_F = \frac{\lambda \cdot \sin(\alpha_1 + \alpha_{-1})}{2} = d$$ => self-imaging of G_1 in G_2

Overlay G_2 and $F(x)$:
Moiré patterns are the same for all λ

Moving diffraction gratings (velocity V_g)



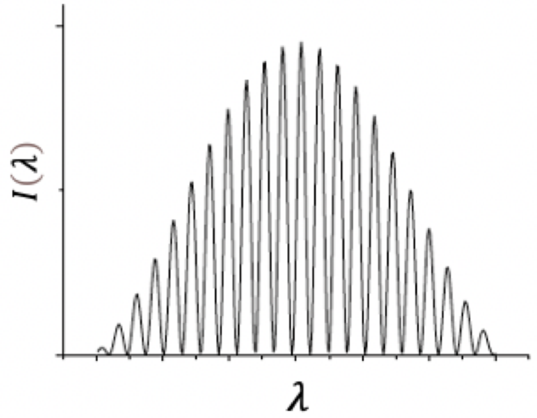
Different neutron wavelength λ:

$\Rightarrow$ different propagation time between G_1 and G_2

$\Rightarrow$ Shift of $F(x)$ relative G_2

$\Rightarrow$ Moiré patterns are different for different λ:

$$I(\lambda) = \frac{I_0(\lambda)}{2}\left\{1 + \cos\left[\frac{2\pi}{d} V_g \frac{m_n}{h} L\lambda\right]\right\}$$



Modulation of outgoing neutron beam spectrum: $f_g = \frac{V_g}{d}$

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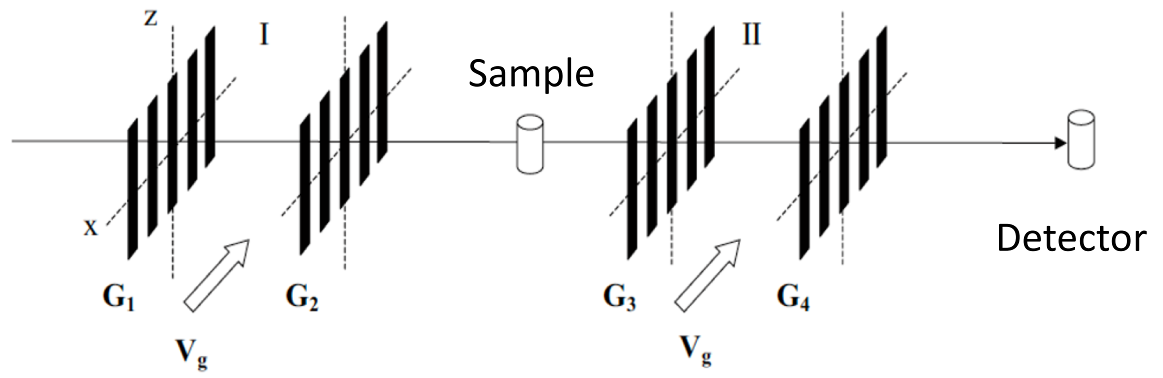
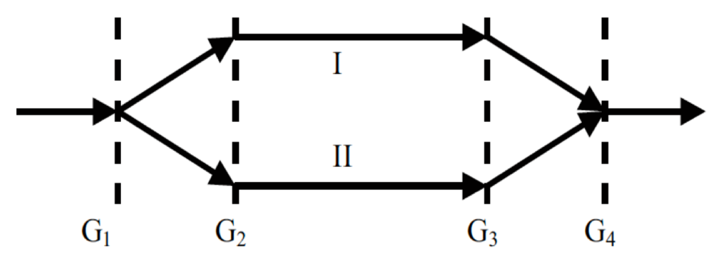
Neutron Speed Echo spectrometer

Two similar arms:

*A. Ioffe, Physica **B283** (2000) 406.*

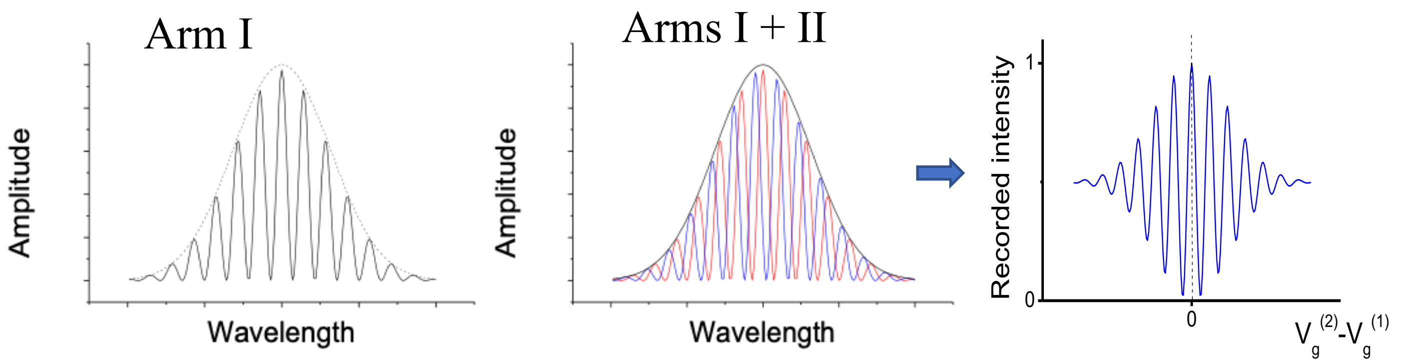




4-grating interferometer: 100% contrast of interference fringes **for non-collimated and non-monochromatic neutron beam**.

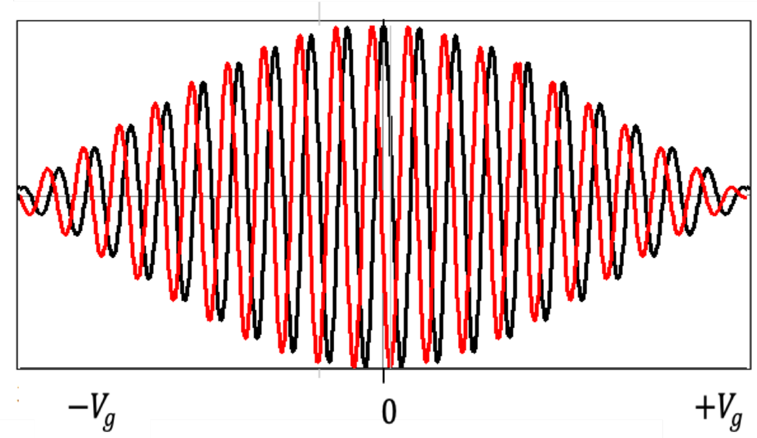
Neutron spetrum modulation

NSPE signal



Inelastic scattering on sample:

$\Delta\lambda$ => shift of NSPE-signal



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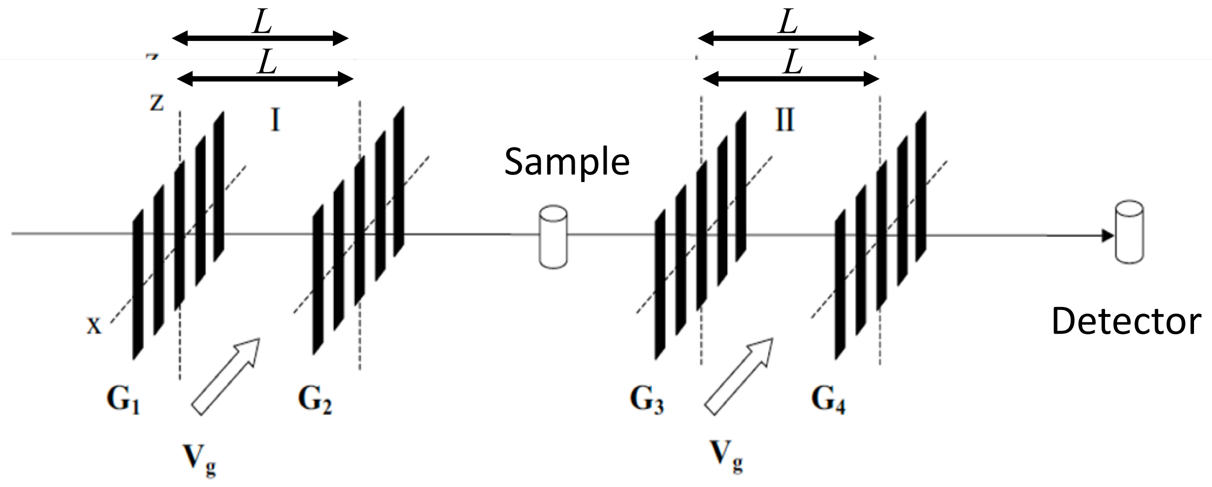
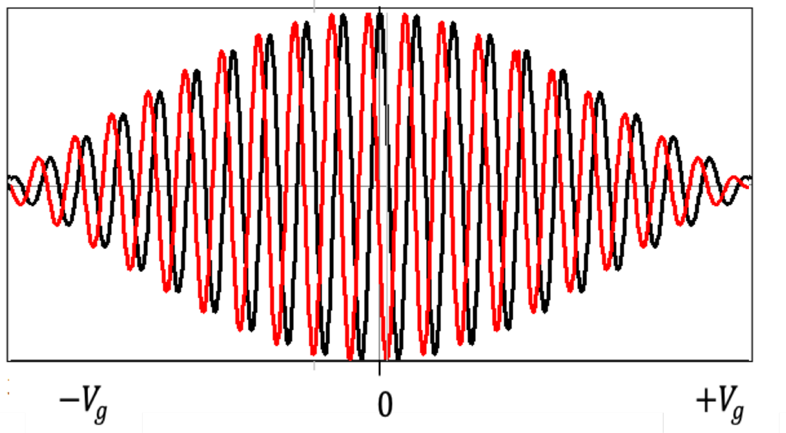
A. Ioffe, Physica ***B283*** *(2000) 406.*

NSPE spectrometer: resolution



Inelastic scattering on sample:

$\Delta\lambda$ => shift of NSPE-signal



We obtain Neutron Spin Echo – like signal, however without the use of neutron spin.

Neutron ~~Spin~~ **Speed** Echo => no polarizazation and pol.analysis

Energy resolution: $\frac{\Delta E_n}{E_n} = 2\frac{\Delta v_n}{v_n} = \frac{d}{\pi L}\frac{v_n}{V_g} r$

r - relative precision of determining the phase shift (≃ 1-2%)

Energy transfer: $\Delta E_n = \frac{h^3}{2\pi\, m_n^2}\frac{d}{L}\frac{1}{V_g} r\, \lambda^{-3}$

Measurable ΔE_n:

λ=10Å => $\Delta E_n / E_n \approx 5\cdot 10^{-7}$, ΔE_n=0.4 neV Similar to NSE

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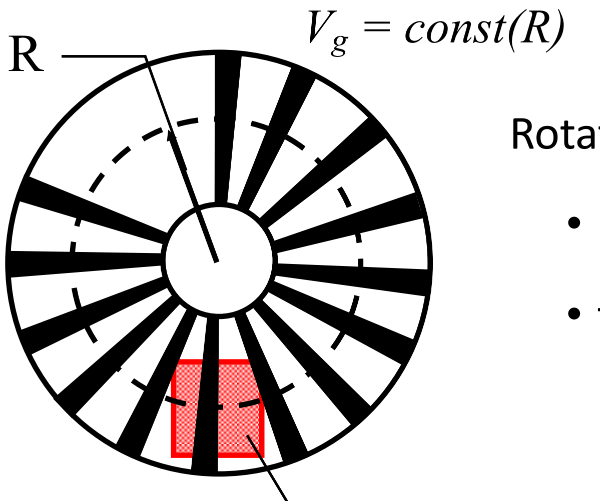
VCN Speed-Echo spectrometer: search for q_n

Radial diffraction grating

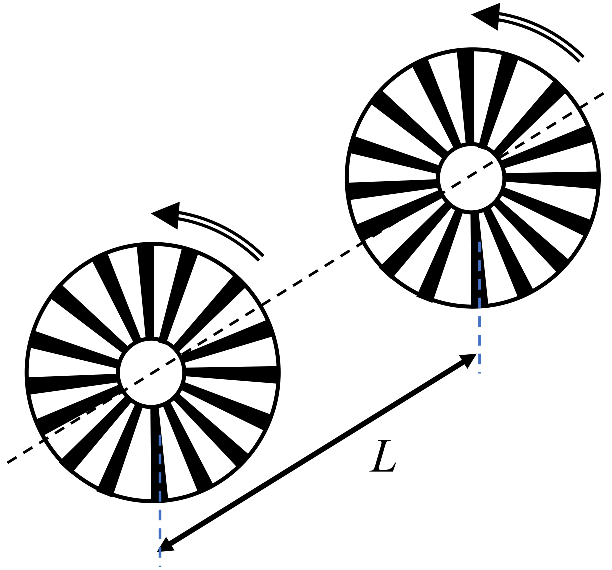


Neutron beam
cross section

Rotating disk (standard chopper):

- up to 18,000 rpm
- frequency f = 300 Hz

f = 300 Hz
R = 30cm
⇓
V_g = 565 m/s



$$\Delta E_n = \frac{h^3}{2\pi \, m_n{}^2} \frac{d}{L} \frac{1}{V_g} r \, \lambda^{-3}$$

d = 3 μm
L = 2 m

λ = 200 Å => $\Delta E_n \geq 3.4 \cdot 10^{-16}$ eV

Neutron charge in electric field:

$\Delta E_n(q_n) = q_n \, E_{el} \, L_{el}$

E_{el} = 60 kV/cm
L_{el} = 2 m

$\Delta E_n(10^{-20}\, e) = 1.2 \cdot 10^{-14}$ eV

30 times better, than present day limit

Conclusion

- Interferometry of Very Cold Neutrons (VCN), relies on diffraction gratings for effectively splitting neutron waves coherently.
- The symmetric 4-grating neutron interferometer fully compensates for aberrations from diffraction gratings and Earth's gravity. This type of interferometer functions independent of source coherence, **accommodating non-monochromatic and non-collimated neutron beams**.
- It holds potential for significantly improving the current experimental limit on neutron charge by two orders of magnitude, down to $3\times 10^{-23}e$. The utilization of holographic (interference) gratings with sub-micrometer periods enhances the performance.
- VCN finds application in Neutron Speed-Echo spectroscopy, facilitating the measurement of extremely small changes in neutron energy. This method also shows potential for investigating neutron charges down to $10^{-23}e$.

Thank you for attention!