Some problems of long-wave neutron optics

Alexander Frank

frank@jinr.ru

UCN workshop,
Almaty, 8 April 2024

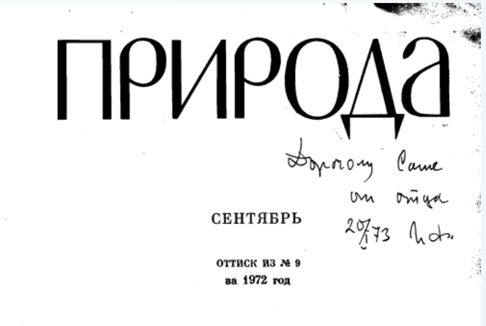
I. Problems of the Neutron Imaging with the micron resolution

- **Neutron microscopy with UCN (History)**
- **Present state of the neutron microscopy**
- **Contrast due to variation of transmittivity**
- **Contrast due to variation of reflectivity**
- **Phase contrast**
- **Contrast due to depolarization**
- **Neutron microscope for VCN with micron resolution. Is it possible ?**

Neutron microscopy with UCN (1972-1990)

Neutron microscopy with UCN (1972-1990)

Motivation – the discovery of UCN with their possibility of their total reflection in a normal falling
Main problems – gravitational aberration and low intensity

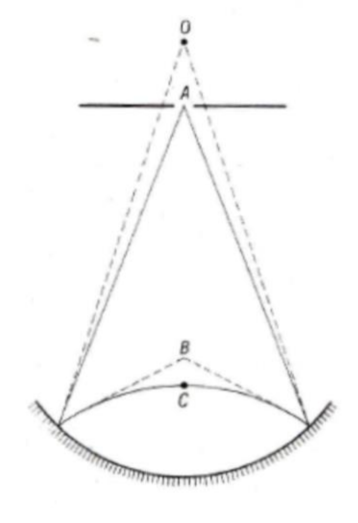


ПРИРОДА

Дорогому Саше
от отца
20/I 73 И.Ф.

СЕНТЯБРЬ

ОТТИСК ИЗ № 9
за 1972 год



Picture from the paper of I.M. Frank
In Sovjet “Priroda” (Nature) journal (1972)

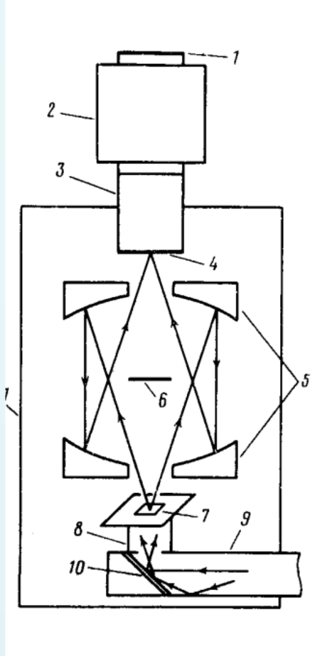
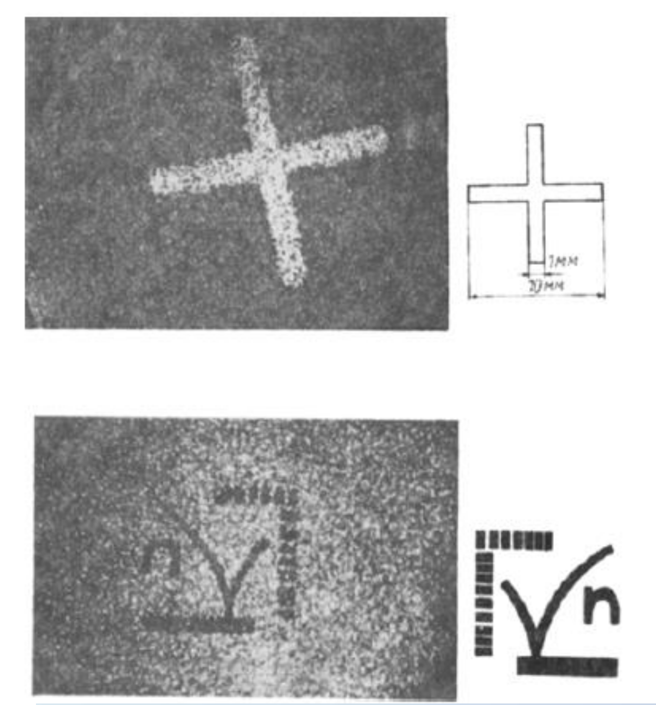


FIG. 1. The apparatus. 1—Photographic film; 2—image converter; 3—optical fiber; 4—scintillator sensitive to ultracold neutrons; 5—mirrors of optics system; 6—shielding from direct “rays”; 7—specimen; 8,9—neutron ducts; 10—auxiliary mirror.



Demonstration of neutron contrast: object and its neutron image

S.S. Arzumanov, S.V. Masalovich, A.N. Strepetov, and A.I.Frank.
JETP Letters, **44**, (1986) 271

Neutron microscope of A. Steyerl

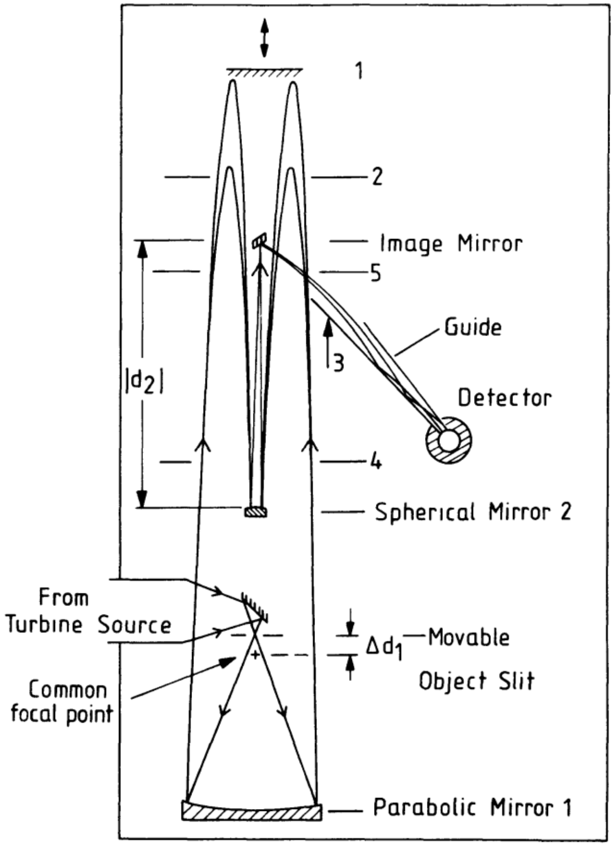


Fig. 1. — Scheme of the two-mirror microscope set-up.

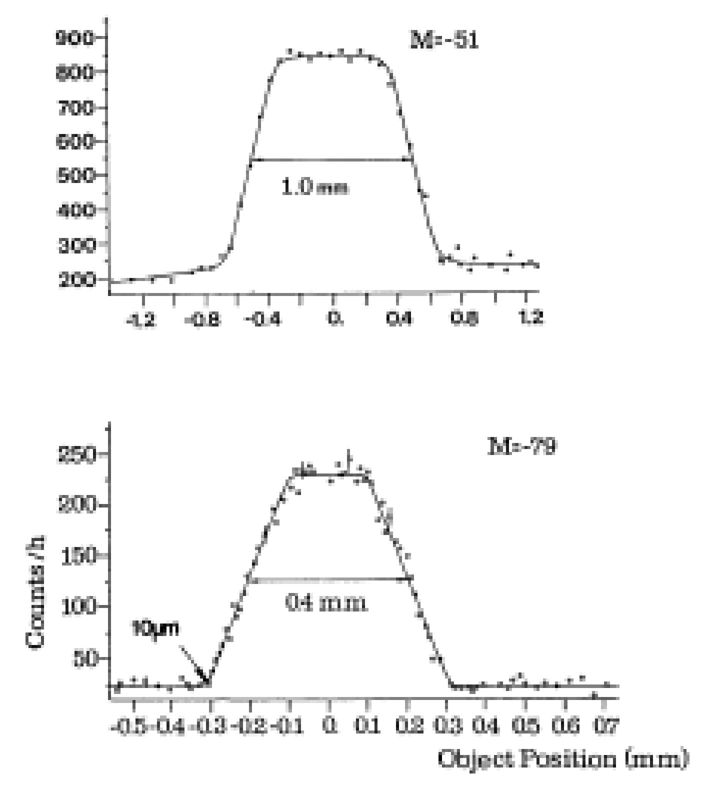


Fig. 3. — Object scans for slits of various size and for different settings of the confocal two-mirror microscope. The edge width $q/|M|$ of the observed trapezoids is determined by the effective image mirror size q = 16.2 mm and by the magnification M. The aberration limit of ~ 10 μm to the resolution may be derived mainly from the corner blurring.

A.Steyerl, W. Drexel, T. Ebisawa et al. Revue Phys. Appl. **23** (1988) 171-180

Horizontal neutron microscope (1990)

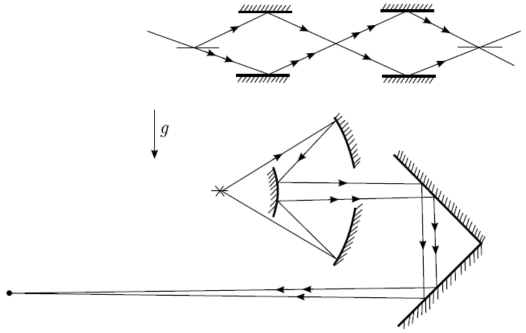
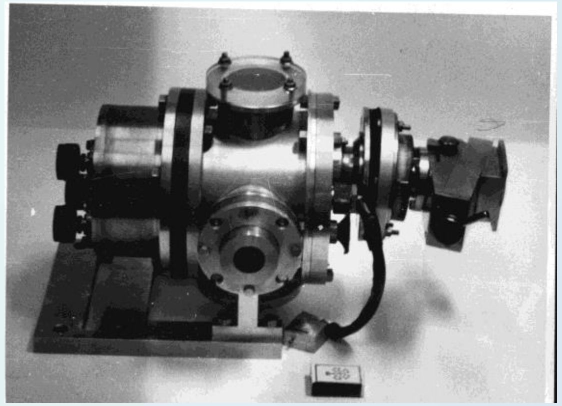


Fig. 5. Two isochronal achromatic devices: two-part neutron interferometer for UCNs in the gravitational field of the Earth and a horizontal neutron microscope with a reversing device



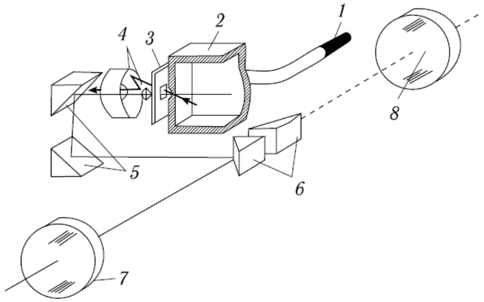


Fig. 6. The horizontal neutron microscope. *1* — optical fiber; *2* — neutron guides; *3* — specimen on the moveable table; *4* — objective mirrors; *5* — reversing devices; *6* — auxiliary mirrors; *7* — optical fiber with UCN-sensitive scintillator; *8* — optical fiber of the optical channel

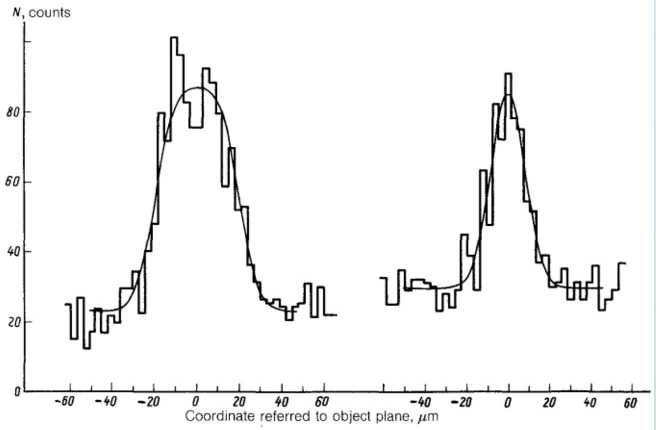


FIG. 2. Neutron count rate versus the coordinate in a cross section of the image of two slits, one 40 μm wide (at the left) and the other 13 μm wide. The smooth curve is a calculated distribution for a Gaussian resolution function with a width at half-maximum of 17 μm.

S.S.Arzumanov, S.V. Masalovich,
A.A. Sabel'nikov, A.N. Strepetov and
A.I. Frank . JETP Letters, **52** (1990), 369

Present state of the neutron imaging with a target resolution in a micron region

1. A grazing angle reflection optics (MIT-NIST) *The aim is 1 μm resolution*

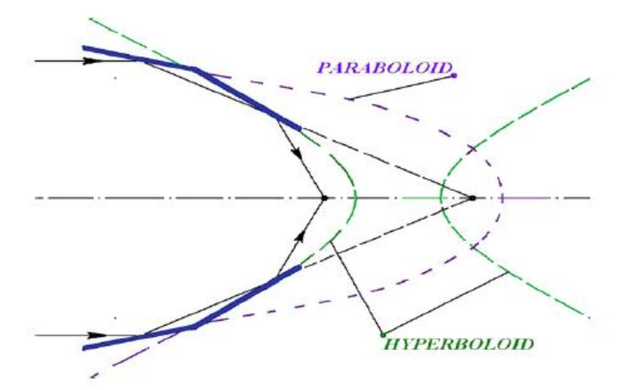
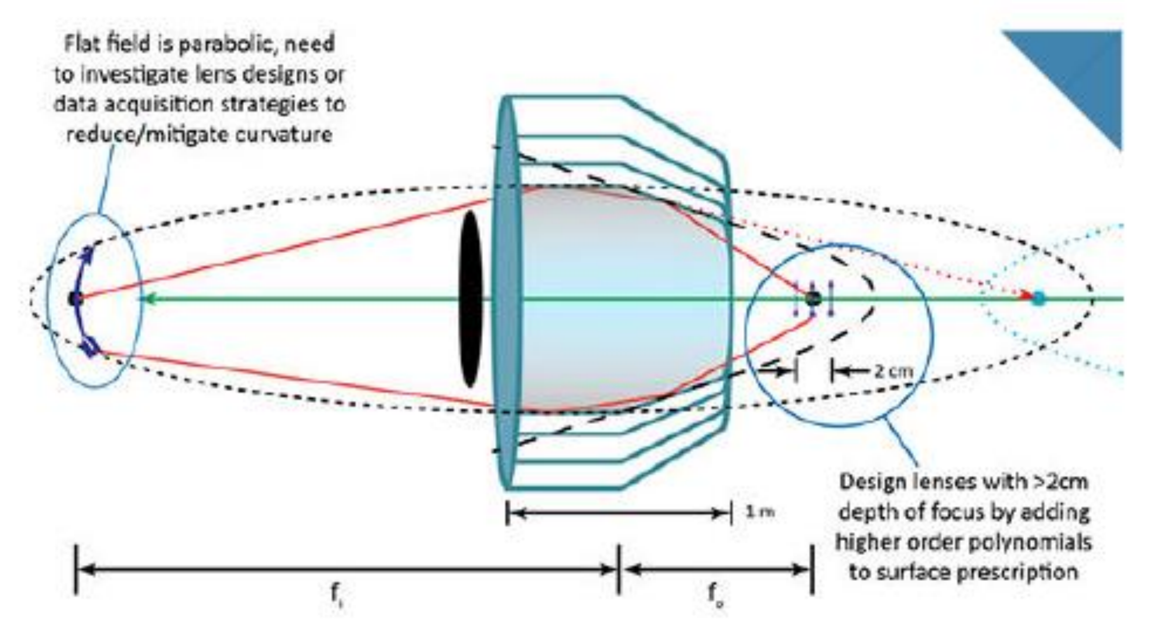


Fig. 3. A Wolter I mirror configuration comprising a paraboloid and a hyperboloid that can focus a quasi-parallel beam to a focus.



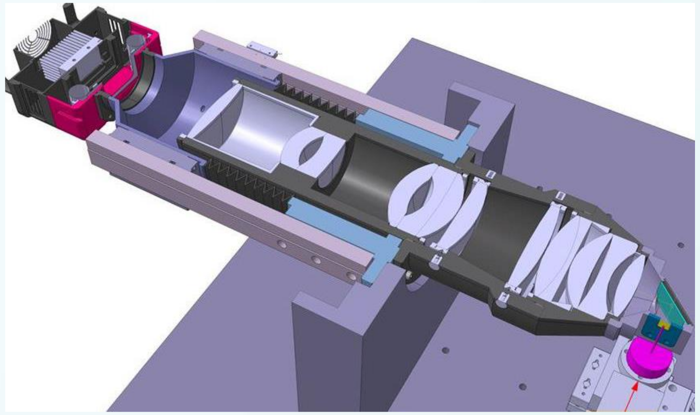
Flat field is parabolic, need to investigate lens designs or data acquisition strategies to reduce/mitigate curvature

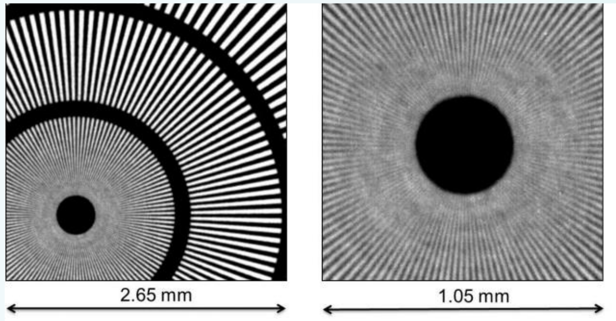
Design lenses with >2cm depth of focus by adding higher order polynomials to surface prescription

D. Liu, D. Hussey, M. V. Gubarev, B. D. Ramsey et.al. Applied Physics Letters **102**, 183508 (2013);
D. S. Hussey, HanWen, Huarui Wu et al. J. Imaging **4**, 50 (2018)

2. Neutron detectors with micron resolution

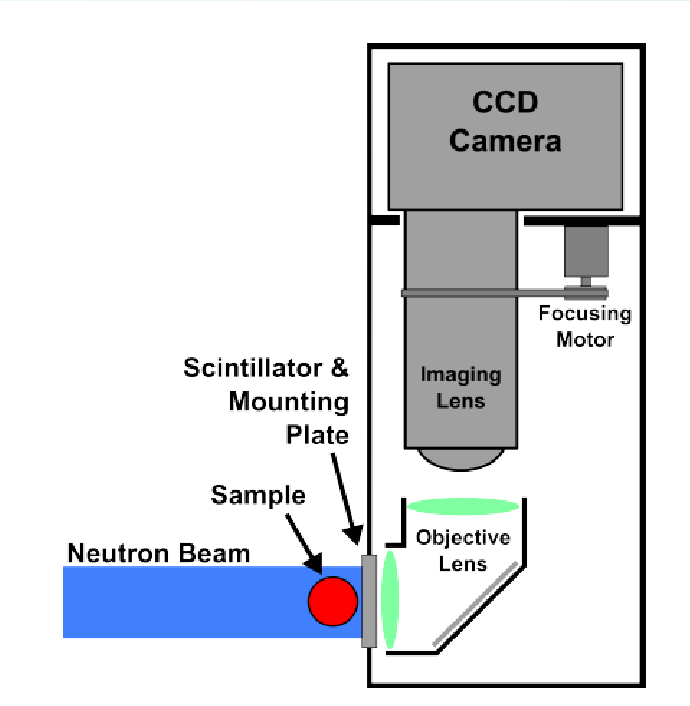
Isotopically-enriched 157Gd2O2S:Tb scintillator





The Fourier ring correlation (FRC)-based spatial resolution of the image is equal to 5.4 µm.

P. Trtik, E. H. Lehmann. J. Phys: Conf. Series 746 (2016) 012004

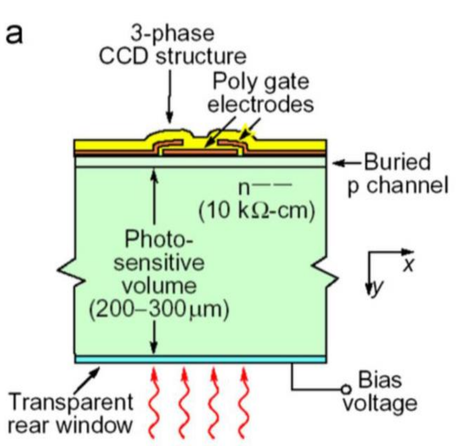


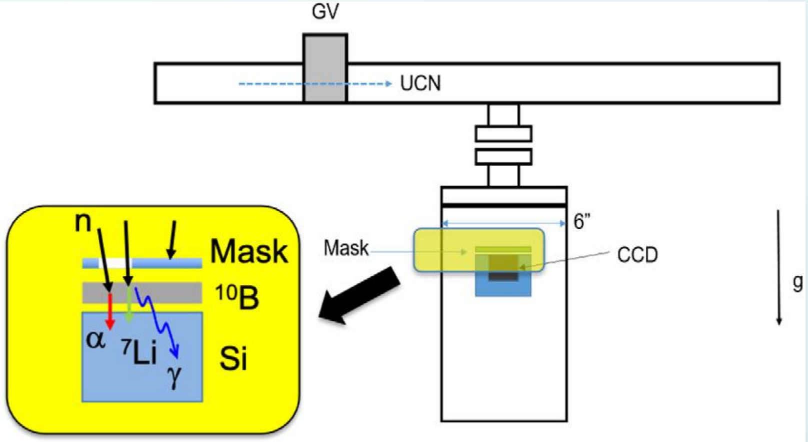
A resolution of 14.8 m with
a field of view of 6 mm 6 mm has been achieved

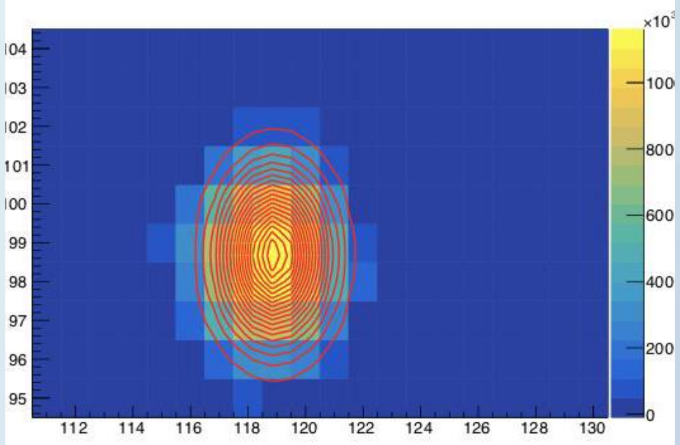
**S.H. Williams,a,1 A. Hilger,a N. Kardjilov et.al.
J. Inst. doi:10.1088/1748-0221/7/02/P02014**

Position-sensitive detectors for UCNs (Los-ALamos collaboration)

K. Kuk , C. Cude-Woods, C.R. Chavez et al. NIM, A 1003 (2021) 165306. (boron CCD camera)







Subpixel resolution

S. Lina, J. K. Baldwin, M. Blatnik et.al. NIM A 1057 (2023) 168769. (Room temp CMOS sensor)

Submicron resolution was obtained!

Neutron radiography with a detector of high resolution requires either

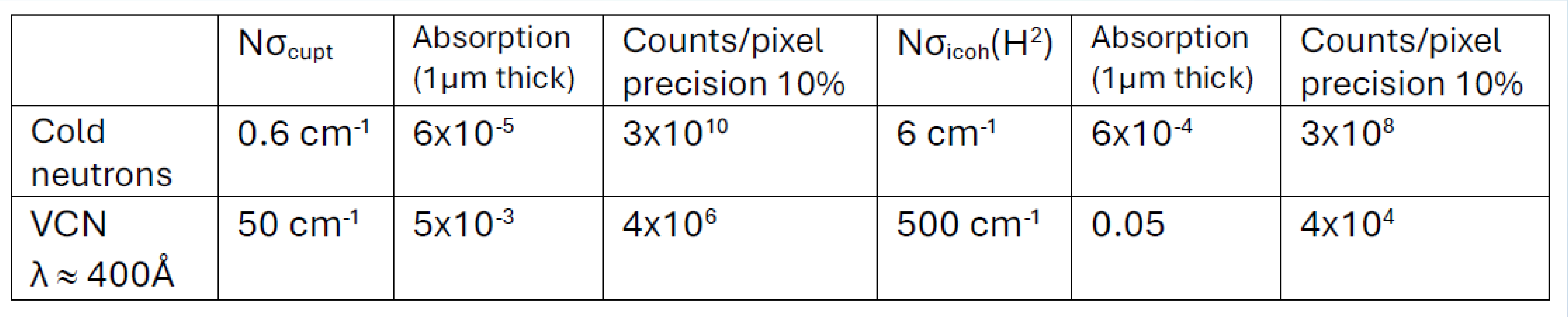
a) High degree of beam collimation, or
b) The close proximity of the object to the detector

That leads to:

a) losses in intensity
b) Limitation of space needed of the manipulation with a sample

The problem of contrast in neutron imaging (Cold neutrons vs VCN)

Contrast due to absorption and incoherent scattering



	$N\sigma_{cupt}$	Absorption (1μm thick)	Counts/pixel precision 10%	$N\sigma_{icoh}(H^2)$	Absorption (1μm thick)	Counts/pixel precision 10%
Cold neutrons	$0.6\ cm^{-1}$	$6x10^{-5}$	$3x10^{10}$	$6\ cm^{-1}$	$6x10^{-4}$	$3x10^{8}$
VCN $\lambda \approx 400Å$	$50\ cm^{-1}$	$5x10^{-3}$	$4x10^{6}$	$500\ cm^{-1}$	0.05	$4x10^{4}$

Using the thermal or cold neutrons it is possible to detect **contrast in transmission** only for matter with **huge cross-section**

2. Contrast due to reflection (VCN).

$V = 10\ \mathrm{m}/s$ $V_b = 4.5\ \mathrm{m}/s$ $\boxed{T = 1 - R \approx 0.85}$ $N \approx 4.5\ \mathrm{x}10^3$

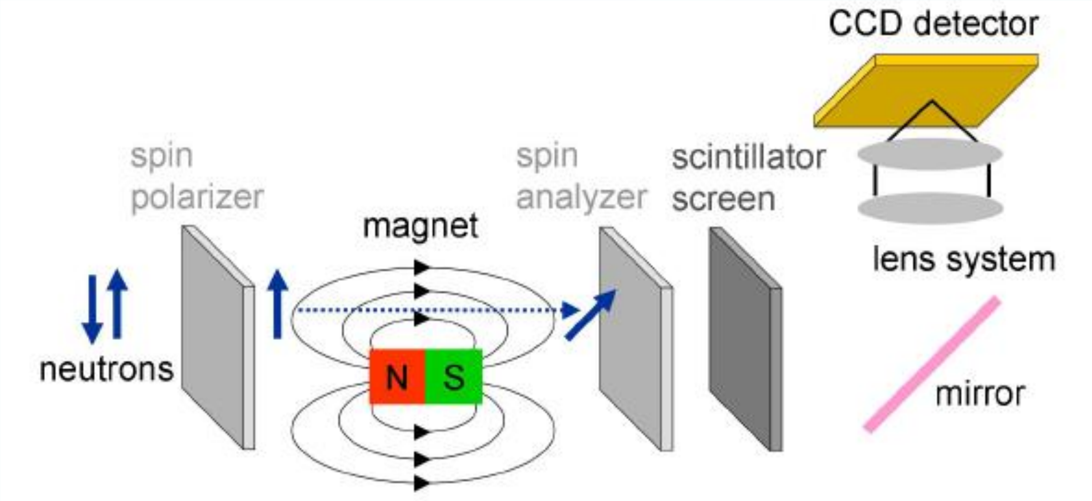
3. Phase contrast. Phase shift due to transmission of 1 μm thick object.

$\varphi = kd\left(1-n\right) \simeq k\frac{V_b^2}{2V^2}$ $\varphi \propto V^{-1}$

$V_b = 4.5\ \mathrm{m}/s$ $V = 1000\ \mathrm{m}/s$ $\varphi \simeq 0.13$

$V = 10\ \mathrm{m}/s$ $\varphi \simeq 13$

Contrast due to depolarization.



From I. Manke, N. Kardjilov et al. (2009)

Remarkable depolarization take place when

$$D \leq \frac{\pi V}{\gamma B}$$

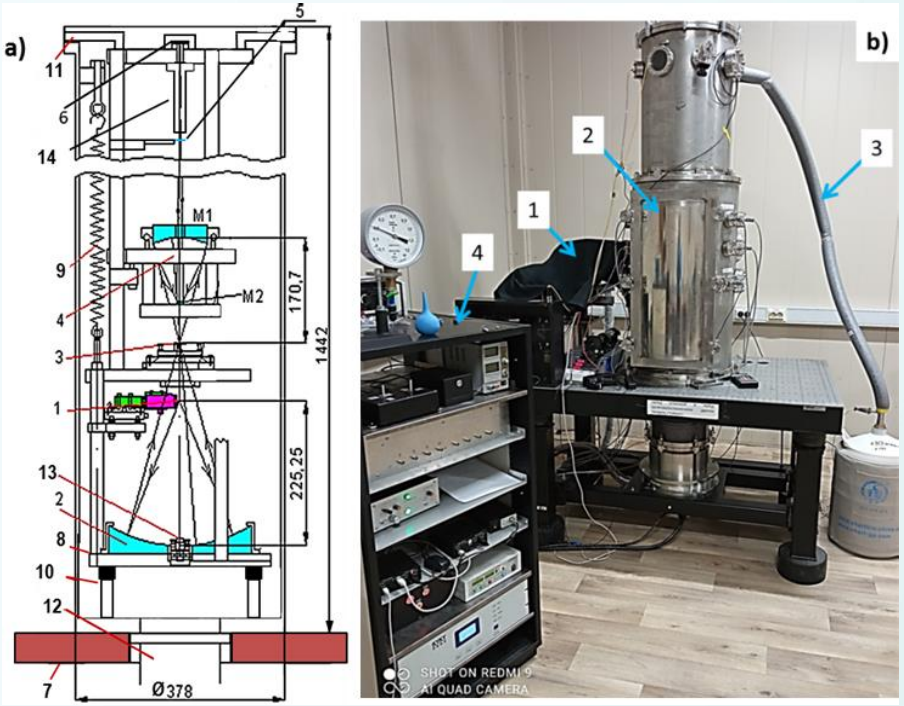
D is the typical size of region with magnetic field B,
γ – gyro-magnetic ratio

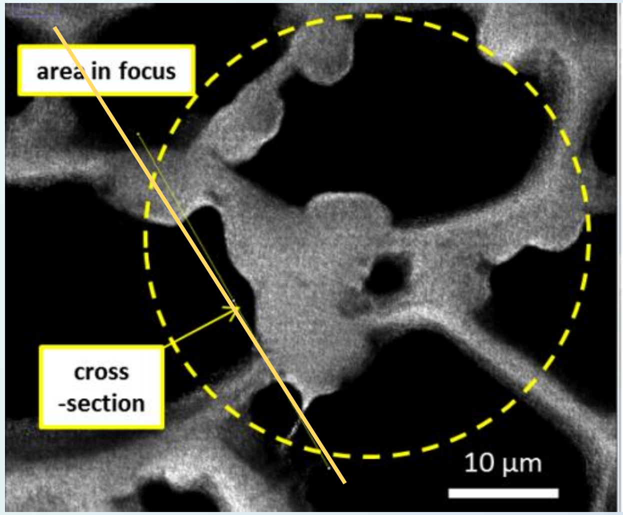
Sensitivity to the magnetic nonuniformity is decreasing with decreasing of velocity

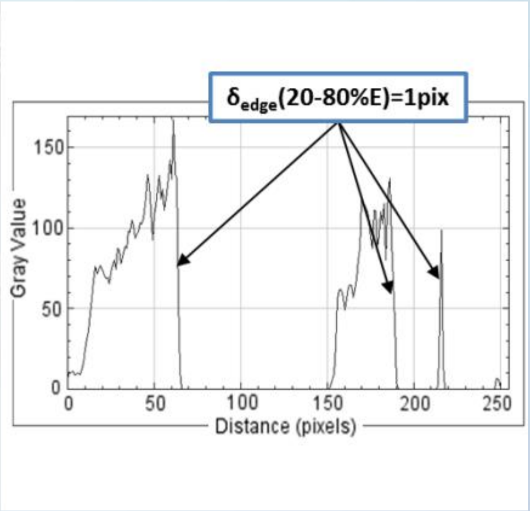
Neutron microscope for VCN with micron resolution. Is it possible?

We can refer to the recent beautiful results in X-ray microscopy with the optic of normal falling

I. V. Malyshev, D. G. Reunov, N. I. Chkhalo et al. Optics Express, 30, (2022) 47567







$$\lambda = 13.84\text{nm} \qquad NA = 0.27 \qquad \delta = 140\text{nm}$$

The counting rate of a neutron microscope with the same numerical aperture being install at a UCN source with a density of 10^(4) n/cm^3 will be 50 counts/day per pixel with an area of 1x1μm^2

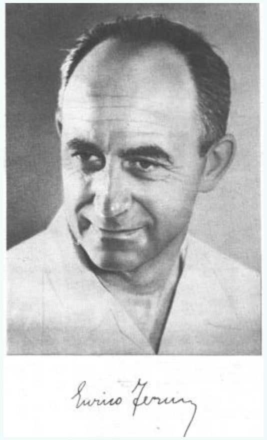
1. **Theoretical base for the neutron microscope with UCN was created in the end of the last century**
2. **Several schemes of the NM were proposed and analyzed**
3. **The activity was stopped due to weak progress of the UCN sources**
4. **The huge progress in technology and UCN sources was achieved during last 20 years**
5. **The X-ray microscope with the submicron resolution is exist**

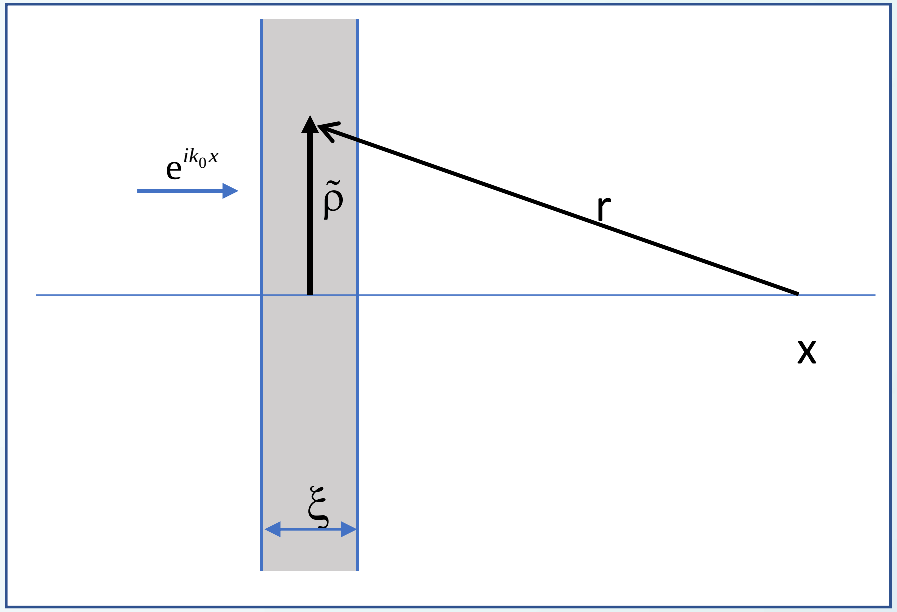
Is it the time to turn back to this problem ?

II. Dispersion Law of the long wavelength neutrons

- **Introduction. Well known relations.**
- **Theoretical predictions for the small corrections to dispersion law of cold neutrons**
- **Specific properties of the Potential-Like Dispersion Law (PLDL)**
- **Experimental approaches to the test of the PLDL validity**
- **Proposal of the alternative approach**

Refraction index and dispersion relation for the neutron waves





E. Fermi, 1944-1950

$$k^2 = k_0^2 - 4\pi\rho b$$

L.Foldy,1945

Potential-like dispersion law

$$\boxed{U = \frac{2\pi\hbar^2}{m} Nb}$$

Effective potential never was used by E.Fermi)

$$\boxed{n^2 = 1 - \frac{2\pi}{k_0^2}\rho b}$$

$$b = b' - ib''$$

Corrections to the dispersion law of neutron waves

$$n^2 = 1 + \frac{4\pi}{k^2}\rho f C, \qquad f = -b + ikb^2 \qquad C = (1-J)^{-1} \approx 1 + J' + J''$$

$$J = Nb\int \exp(i\mathbf{k}\cdot\mathbf{r})G(\mathbf{r})\left[1-\mathbf{g}(\mathbf{r})\right]d\mathbf{r} \qquad G(\mathrm{r}) = \exp(ikr)/r$$

$$C' \approx 1 + 2\pi\rho b' a^2$$

$$C'' \approx \pi\rho b' k a^3$$

$\boldsymbol{ka} \to 0$

V.F. Sears,1982

a – *interatomic distance*

$$\boxed{n^2 = 1 - \frac{4\pi\rho b}{k_0^2}\left[1 + \frac{4\pi\rho b}{k_0^2}\int \mathrm{e}^{ik_0x}\sin(nk_0x)\left[g(x/k_0)-1\right]dx\right]^{-1}}$$

M. Warner & J.E.Gubernatis,1985

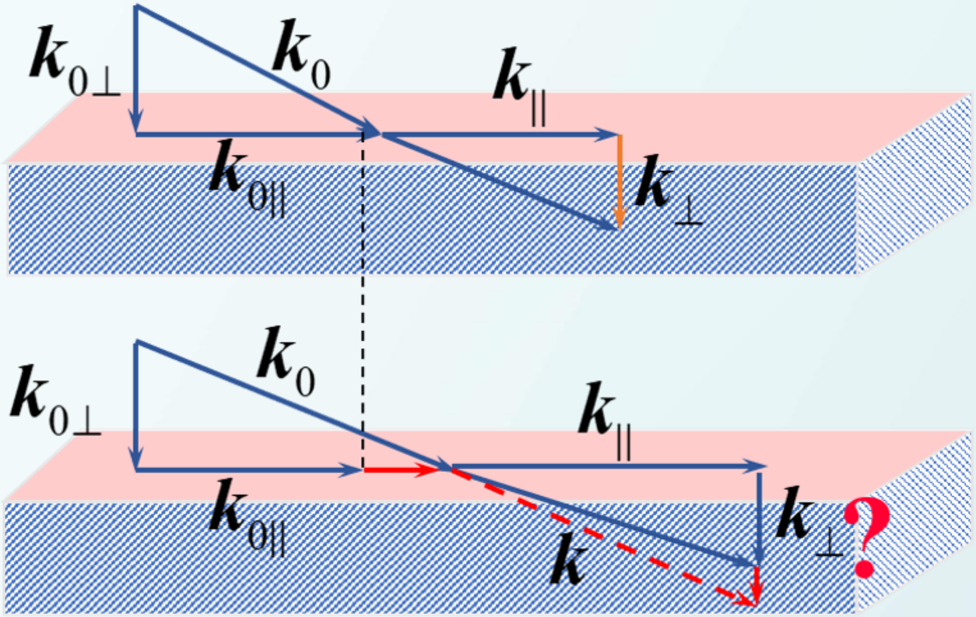
$\boldsymbol{k}_0 \le 4\pi\rho\boldsymbol{ba}$ (Super Slow Neutrons, SSN)

$$\xcancel{\boldsymbol{n}^2 = 1 - \frac{4\rho N}{\boldsymbol{k}_0^2}\boldsymbol{b}}$$

V.G.Nosov & A.I.Frank, 1991

Unknown dispersion law for SSN and small corrections to the UCN dispersion law !

Specific feature of the potential-like dispersion law



$$k_{II}^2 = k_{0II}^2$$

$$k^2 = k_0^2 - \chi^2;$$

$$\chi^2 = 4\pi\rho b$$

$$k_\perp^2 = k_{0\perp}^2 - \chi^2;$$

$$b = const$$

I.M.Frank, 1974,
A,G.Klein, S.A.Werner, 1983

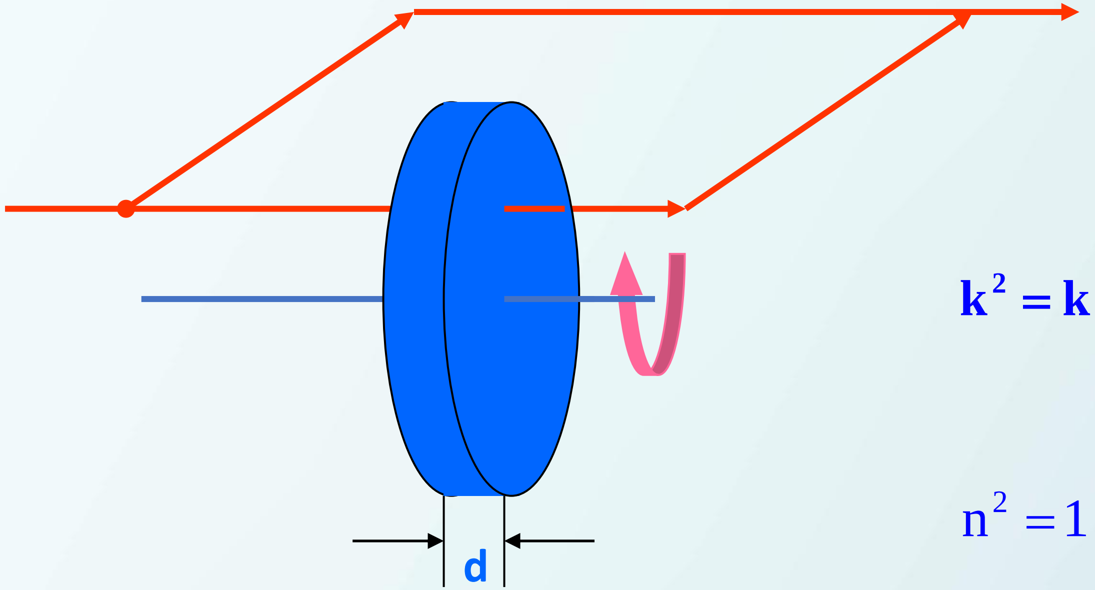
$$k^2 = k_0^2 - \chi^2 + \varepsilon\left(k_0^2\right);$$

$$\chi^2 = 4\pi\rho b$$

$$k_\perp^2 = k_{0\perp}^2 - \chi^2 + \varepsilon\left(k_0^2\right);$$

Strategy of the test experiment – looking for $k_\perp$ with variation of $k_\parallel$ at $k_{0\perp} = \text{const}$

Fizeau-type experiment with UCN



Phase shift $\boxed{\varphi = k(1-n)d}$

$$\mathbf{k^2 = k_0^2 - k_b^2} \qquad \mathbf{k^2 = k_0^2 - \left[k_b^2 + \varepsilon(k_0^2)\right]}$$

$$n^2 = 1 - \frac{k_b^2}{k_0^2} \qquad n^2 = 1 - \frac{\left[k_{lv}^2 + \varepsilon\left(k_0^2\right)\right]}{k_0^2}$$

Experiments for the test the validity of the potential-like dispersion law

Thermal neutrons

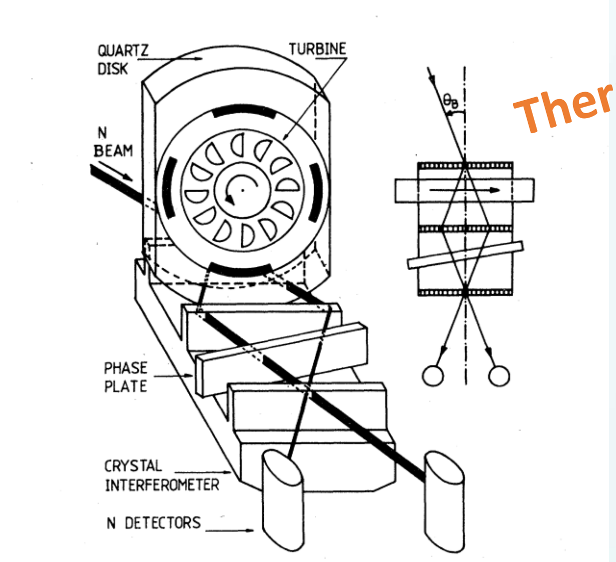


FIG. 1. Overall layout of neutron interferometer showing rotating quartz disk. Inset: schematic of neutron interferometer.

M. Arif, H. Kaiser, S.A.Werner et al.
Phys. Rev.A 31 (1985) 1203

Phase shift was due to rotation was not observed due to not enough sensitivity of the experiment

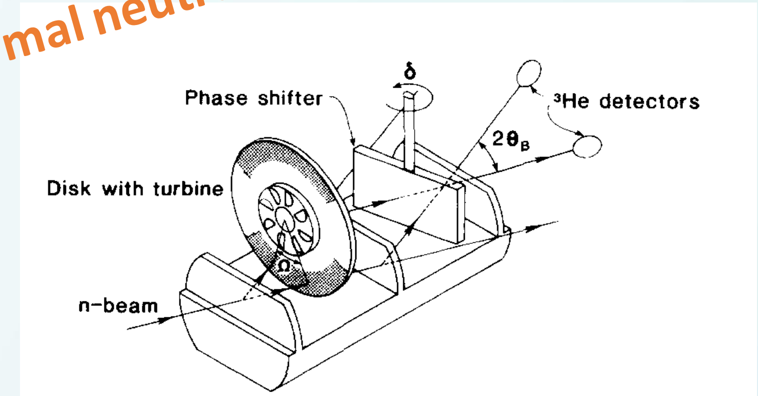
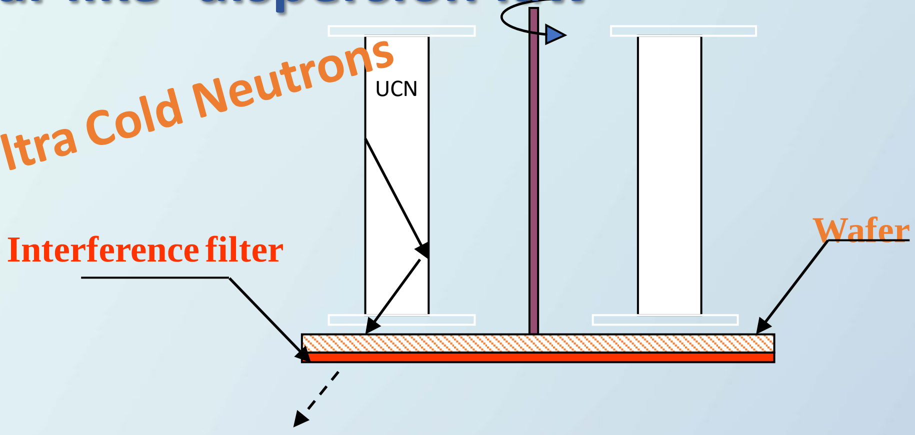


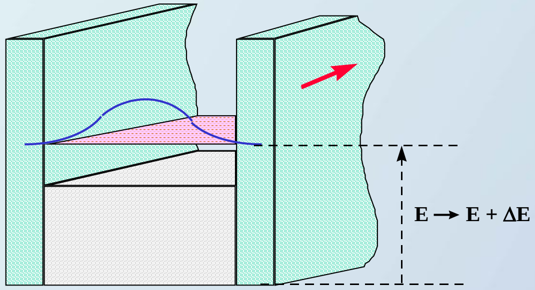
Fig. 2. Overall layout of the neutron interferometer showing the placement of the aluminum disk. The shaded segments on the disk correspond to the positions of the samarium foils.

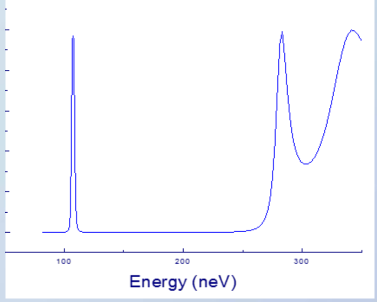
M. Arif, H. Kaiser, R. Clothier et al.
Physica B 151 (1988) 63-67

Prove of the principle. Observation of the effect for the matter with resonant cross-section $b \neq \text{const}$

Ultra Cold Neutrons



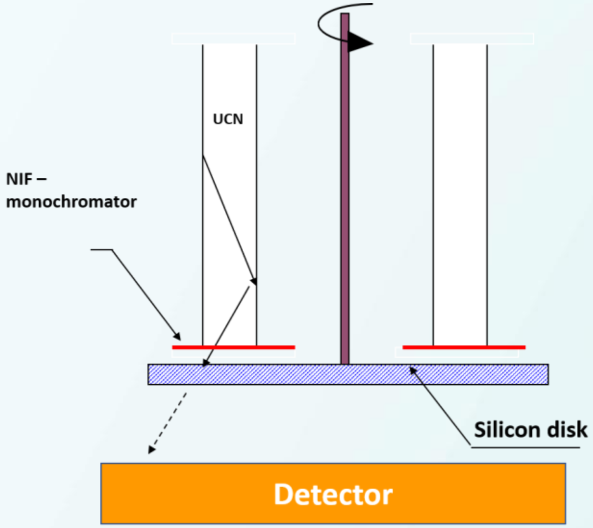


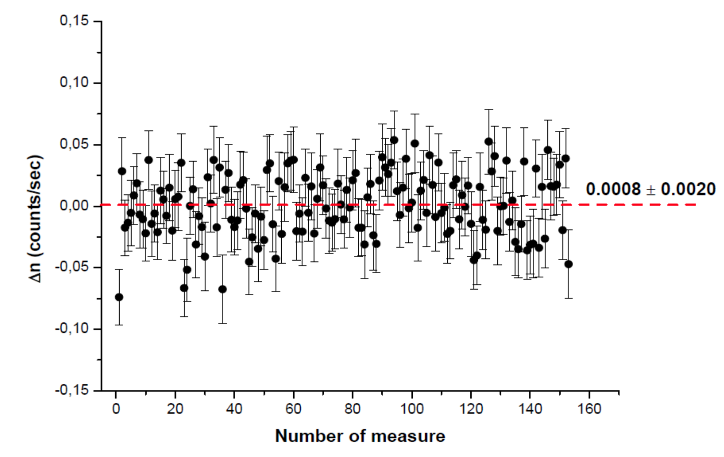


I.V. Bondarenko, A. V. Krasnoperov, A. I. Frank at al.
JETP Letters, 67, (1998) 786.

Systematic effect was found

Experiments for the test the validity of the potential-like dispersion law for UCN





$$v_x = 6 \iff 36\ m/s$$

$$\Delta n / n = (0.6 \pm 1.4) \times 10^{-3}$$

Transmittivity depends on two parameters: real and imaginary parts of “potential”

$$U = V - iW = \frac{2\pi\hbar^2}{m} N\left(1 + J' + iJ''\right)\left(b' - ib''\right)$$

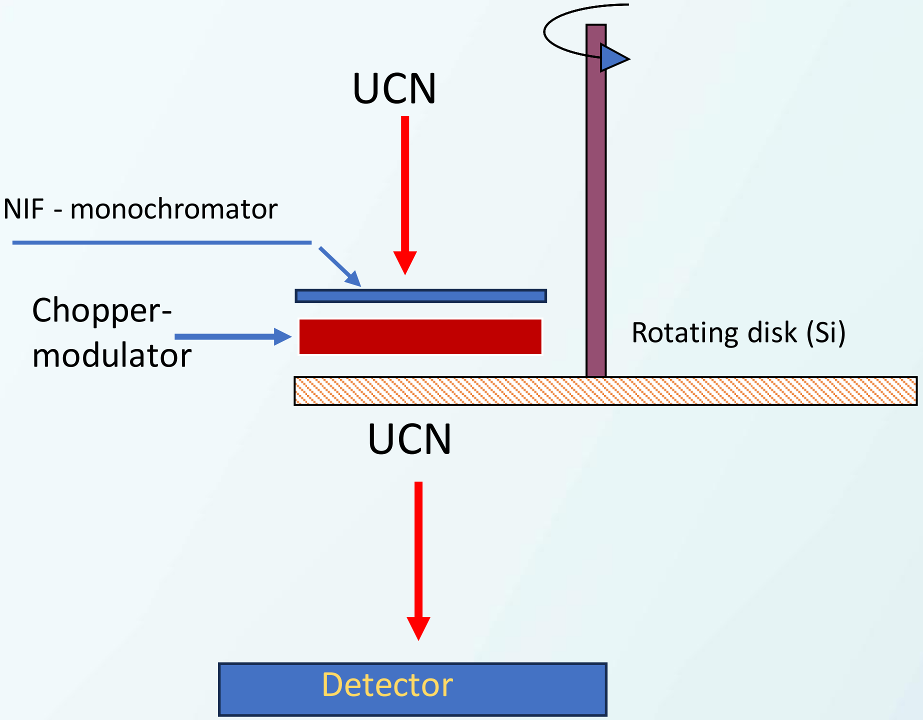
$$\delta J' = \ \leq 3\times10^{-3}\ if\ \ \delta W = 0$$

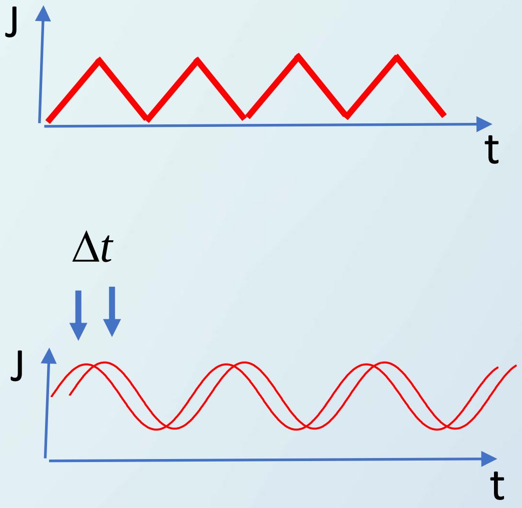
$$\delta J'' = \ \leq 3\times10^{-8}\ if\ \ \delta V = 0$$

G.V. Kulin et al, 2014
A.I. Frank, 2016

Proposed approach for the test the of validity of the potential-like dispersion law for UCN

It is possible to look not for the wave number inside the matter, which is complex, but for the velocity, which is real value!





UCN 110 neV
The estimated delay time due to refraction in silicon with thickness of 2mm is about 20 ms

Conclusion

The question of the validity of the potential - like dispersion law for UCN still remains open and this situation cannot be considered acceptable

Thank you for you attention!